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Physics Laboratory Exercises

based on Polish textbooks written by
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List of symbols

Greek Symbols

$[\alpha]$	the proper torsional ability (ex.306)	
α	the angle of the torsion of the polarization plane by the sugar-in-water solutions (ex.306)	
α	the linear expansion coefficient of solids	K^{-1}
α	the thermoelectric coefficient (ex.207)	V/K
$\cos\phi$	the cosine of the losses angle (ex. 202)	
Δa_{reg}	uncertainty of the slope factor of linear regression	
Δb_{reg}	uncertainty of the b factor of linear regression	
η	the luminous efficiency (ex.307)	
η	the transformer efficiency in [%] (ex.202)	
κ	ratio of specific heat at constant pressure to specific heat at constant volume	
λ	wavelength of light or sound	m, cm, nm, Ă...
μ	molar mass	kg/mol
ω	circular frequency	rd/s
Φ	the luminous flux (ex.307)	
ϕ_0	initial phase	rd or $^\circ$
Φ_C	the total luminous flux (ex.307)	
ρ	density	g/cm^3 kg/m^3
σ	The standard deviation of any measurement	
σ_s	The standard deviation of the arithmetic mean	
ε	the deviation of the measurement value from the arithmetic mean	
ε	the electromotive force (EMF) (ex.201)	V
$\varphi(x-x_s)$	the Gauss function of the distribution of errors	
ϑ	the angle of light deflection passing thru the diffraction grating (ex.303)	

Roman Symbols

A	amplitude	
a_l	the position a_l of the lower scratch of plate ex.301	
a_u	the position a_l of the upper scratch of plate ex.301	
a_{reg}	the slope factor of linear regression	
b_{reg}	the b factor of linear regression	
C	the capacitance of a capacitor (ex.201)	F
c	concentration of the sugar-in-water solution (ex.306)	
D	steering torque pendulum springs	$kg\ m^2\ s^{-2}$
d	actual thickness of plate ex.301	
d	the diffraction grating constant (ex.303)	

dt	differential of time	
E	Young's modulus	GPa
E	the illuminance (ex.307)	
E_F	the Fermi energy of electron levels (ex.207)	eV
E_n, E_m	energies of electron on nth and mth orbital (ex.304)	
E_{F0}	the Fermi energy at 0 K (ex.207)	eV
f	frequency	Hz
f	the focal of the lens or set of lenses (ex.301)	
h	Planck constant (ex. 205, 304 - see also B.1)	J s
h	apparent thickness of plate ex.301	
i	a current in the circuit (ex.201)	A
i	current	A
i	the distance from the lens to image (ex.302)	
i	the temporary current value (ex.202)	A
I_0	moment of inertia respect to the axis of symmetry	kg m^2
I_S	the luminous intensity (ex.307)	
j_A,j_B	the density of the thermocouple current (ex.207)	A/m^2
K	the transformer's transmission (ex.202)	
k	wave number	cm^{-1}
l	length	m
l_r	the reduced length of pendulum (ex.102)	m
M	mass, for example disk mass ex.104	kg
m	mass	kg
n	amount of mole	
n	the refractive index (ex.301,305)	
o	the distance from the lens to object (ex.302)	
P	The probability of obtaining the result in the range (x_1, x_2)	
P	power	W
P	the electrical power (ex.307)	
p	pressure	Pa Tr atm
Q	an electric charge (ex.201)	C
Q	the Joule-Lenz heat (ex.207)	J
R	a resistance of the resistor (ex.201)	Ω
R	gas constant	J mol^{-1} K^{-1}
R	radius, for example disk radius ex.104	m
R	resistance	Ω
r	distance from axis of rotation, radius of object	m
T	period [1/s] or temperature in	K
t	time	s
T_f	a periof of the physical pendulum (ex.102)	s
T_m	a period of the mathematical pendulum (ex.102)	s
U	a potential difference - voltage (ex.201)	V
U	voltage	V

u	the temporary voltage (ex.202)	V
V	volume	m^3 cm^3
v	the speed of sound, vehicle, bullet, any object e.t.c.	m/s
V_k	the potential difference (ex.207)	V
W_A,W_B	the output work of the electrons (ex.207)	J or eV

1. Introduction

This *pdf* document is prepared as a summary of the most important information on physics laboratory exercises in English language. Is the result of translating selected fragments of the script written by Stanisław Szuba [1, 2] and Krzysztof Łapsa [3]. Also consist of translation the notes available on *the website* of the Faculty of Technical Physics at Poznan University of Technology.

In order to fully prepare for the exercises, familiarize yourself with the keywords and learn from the supplementary literature [4–7].

This document will be systematically developed and made available to students performing exercises in physics laboratory in English.

Good luck

P. G.

2. Analysis of the measurements results

1. Physical measurements

Experimental determination of physical quantity is usually a complex activity in which three stages can be distinguished:

- making direct measurements,
- calculation of the searched value on the basis of a known functional relationship between this quantity and the quantities measured directly and
- assessment of the error that is assigned to the determined value.

Only a few quantities can be measured directly with the appropriate instruments. These include basic quantities: time, length, mass and current. The measurement consists in comparing a given physical quantity with a standard of this quantity taken as a unit. A number of other sizes can also be read directly on the scale of the instrument; these measurements are also included in the direct, although the indicated value is the result of a different size measurement and appropriate calculation. For example, a (analog) voltmeter is a device that measures current directly, and the voltage is calculated from Ohm's law $U = Ri$ (R is a known internal resistance) and we read this result on the scale of the device. Measured values are called *simple quantities.*

In order to determine the composite quantity, we must measure several simple quantities and then apply the appropriate formula combining the simple quantities with the searched value.

The measurement of a given physical quantity can be a complex or direct measurement, depending on the instruments used. As an example, let's consider the measurement of electric power. If we have an ammeter and a voltmeter, then we measure two simple quantities - current (i) and voltage (U), and then calculate the power from the relationship $P = Ui$. In this situation, P is a composite quantity. Otherwise, we can use a watt meter that will indicate the power of the current directly on the scale of the instrument - then P is a simple quantity.

We save the results of direct measurements as well as the results of calculations in tables. When planning a table, we need to predict which quantities will be measured directly and which will be calculated. *It is incorrect to enter in the table the results obtained by simple calculations in memory, bypassing the primary readings.* For example, for measuring the period of oscillation of the pendulum, we foresee three headings: 1 - number of vibrations (n), 2 - time of n vibrations (t), 3 - period of vibrations (T), and not only the last column in which we would enter the quotient calculated in the memory total time and number of vibrations. First of all, the table must reflect direct measurements. Similarly, save each measurement result repeated many times, not just the average value.

Table 2.1. Sample measurement table

Frequency f [Hz]	Position 1 x_1 [cm]	Position 2 x_2 [cm]	Position 3 x_3 [cm]	$x_2 - x_1$ [cm]	$x_3 - x_2$ [cm]
3505	21.4	35.8	47.6	14.4	11.8
5220	18.0	25.2	33.7	7.2	8.5

2. Calculations and charts

General rules

The measured simple quantities allow to determine, depending on the nature of the measurement, *universal constants* (e.g. elementary charge, Planck constant), *material constants* (e.g. modulus of elasticity, refractive index) or determine the *functional dependence* of one physical quantity on the other (e.g. conductivity from temperature, lighting from a distance).

Diagrams

In the case of calculating constants, both universal and material, the second stage of the experiment consists in inserting the results of direct measurements into the correct formula and making calculations. The final result is the nominated number expressed in units of the determined physical quantity.

When the goal of the experiment is to find a relationship between two physical quantities, then the result is usually presented in graphical form. Measurements then consist in a deliberate change of one quantity, e.g. x, and finding corresponding values of the other quantity, e.g. y. The graphical image of the relationship is *the graph of the function* $y = f(x)$. Preparation of the correct chart requires compliance with the following rules,

1. *The abscissa* (horizontal) represents the independent variable and *the ordinate axis* represents the dependent variable. Near the axis, enter the names of the appropriate knowledge and unit. The length of both axes should be approximately the same - 10 - 20 cm. Then the plot area will be close to square. The longer the axis, the more details the graph contains.
2. *The scale* is applied to the coordinate axis in such a way that the range measures he had almost the entire length of the axis. At the beginning of the axis or near it, we place *scale divisions* corresponding to the smallest measured values - they do not have to be zero values. The largest measured values should be near the end of the

axis. The scale divisions should be chosen so that any value of the measured value can be easily found on the axis. This requirement will be met when we apply the rule:

$$1 \text{ cm axis} = (1 \text{ or } 2 \text{ or } 5) \cdot 10^n \text{ units} \tag{2.1}$$

where n is an integer. After determining the scale division, the length of the axis can be changed change compared to originally planned.

3. *The tick labels* of the axis are created by adding values for some main ticks. Along the axis there should be 3-6 values described, spaced at regular intervals. It is incorrect to mark the measurement values on the axes.

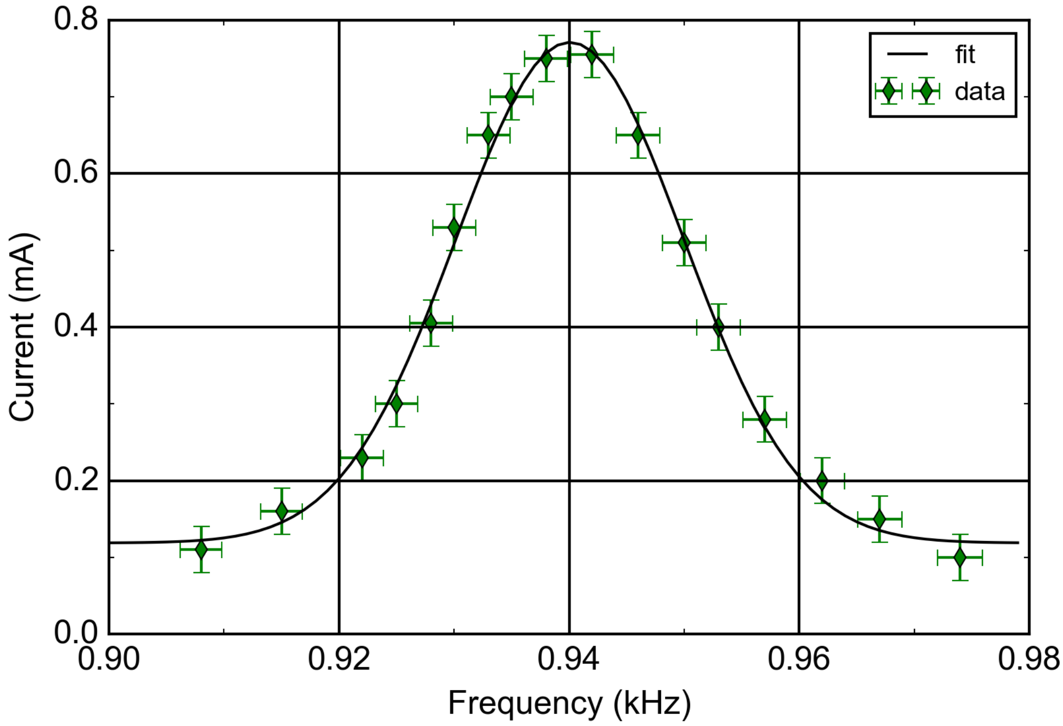


Figure 2.1. An example of a chart in linear coordinates

4. Mark the *measuring points* with crosses, circles or other geometrical figures, but not with dots that are hardly visible. When we place several curves on one sheet, we mark the points belonging to each of them in a different way. The measuring point itself should be located in the center of the mark. The number of points that should be used when preparing the chart must not be too small - at least 10-15 points. If the curve shows a maximum, minimum or inflection point, then near these places, the measuring points should be concentrated, because this allows the curve to run correctly.
5. *The curve* reflecting the relationship should be smooth, i.e. without sharp bends and local extremes. Usually it does not run through all measuring points, but in such a

way that the number of points lying on both sides is approximately equal and their random distribution.

EXAMPLE

We draw the resonance curve of the vibrating circuit, i.e. the dependence of the AC current on the frequency (Fig. 2.1). Frequency range: 0.906 $<\omega<$0.975 MHz. We assume values for the beginning of the $\omega = 0.9$ MHz system, for the end - *omega* = 0.98 MHz, axis length - 10 cm. Then the 1 cm scale corresponds to 0.008 MHz. The closest value satisfying the condition (eq. 2.1) is 0.01 MHz / cm ($1 \cdot 10^{-2}$). After taking this value, the length of the abscissa is 8 cm. Current range $0 < i < 0.8$ mA We assume the value for the beginning of the system $i = 0$, for the end of the axis - $i = 0.8$ mA, the length of the axis - 8 cm.Then the scale with a length of 1 cm corresponds to the current by $i = 0.1$ mA. axes the chart covers the entire surface and is most readable.

The axes constants in Fig. 2.1 are linear, i.e. the distances between points on the axis are directly proportional to the increases in the size represented by this axis. Graphs on a linear scale are used when we graphically present a linear relationship or an unknown non-linear relationship with a range of changes within one row.

Physical processes, including those we encounter in the laboratory, are often described by nonlinear functions, and the purpose of the measurement can be to check the nature of the function or to determine a certain quantity occurring in a nonlinear relationship. In these cases, we usually use charts with *non-linear scales.*

For example, in a uniformly accelerated motion graph of the path (S) In function of time (1) is a parabola described by the function $s = v_0 t + 1/2at^2$. To check whether the tested motion is really uniformly accelerated, we prepare a graph in the coordinates: $y = s$, $x = t^2$. If the graph is a straight line, it means that the equation has been met, and therefore the uniform accelerated motion is confirmed. A graph that deviates from the straight line indicates that the traffic is of a different type,

As a second example, we consider the dependence of conductivity (σ) on (T) for semiconductors

$$\sigma = \sigma_0 e^{-\frac{E}{k_B T}} \tag{2.2}$$

In order to check whether the measurement points obtained meet the equation (2.2), we draw a graph in coordinates in which the above equation would be a straight line. You can see that these coordinates are:

$$y = ln(\sigma/\sigma_0), \qquad x = 1/T\,. \tag{2.3}$$

After logging in the equation (2.2) and performing substitutions (2.3), we get the relationship

$$y = -\frac{E}{k_B T} x\,, \tag{2.4}$$

whose graph is a straight line with a slope factor of $-E/2k_B$ (Fig. 2.2).

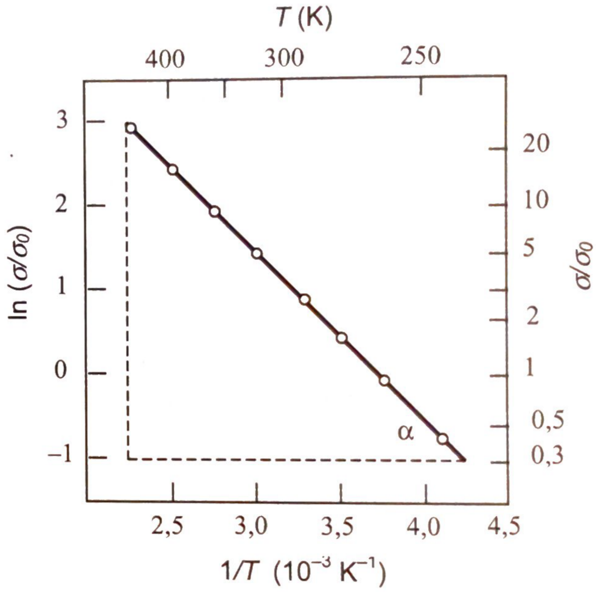


Figure 2.2. An example of a chart in nonlinear coordinates

If the experimental plot in this coordinate system is a straight line, it confirms that the conductivity changes with the temperature according to equation (2.2). Measurement of the *slope coefficient* on the graph also allows to determine additional quantities in the described case, e.g. E (when k is known).

It should be noted that the slope coefficient is not a tangent of the geometrically measured slope, but in general the nominative value is derived from the ratio of coordinates increments $\Delta y/\Delta x$. It can easily be seen that the slope factor in Figure 2.2 is –1430 K.

The description of the chart is often supplemented by the scale of the basic size (with uneven divisions) plotted on the auxiliary axes (σ and T in Fig. 2.2). When preparing data for the chart in nonlinear coordinates, remember to provide in the table for the calculated functions of the measured values, e.g. $\ln\sigma$, $1/T$.

Linear regression

Suppose we measure two dependent amounts of x, y. We place the measuring points determined by appropriate pairs of values x_1, y_1, ...x_i, y_i on the chart and we want to draw a continuous line, which is to show the real relationship between y and x in the whole range of variability x. Because we know that each of the measuring points is affected by a certain error, there is no point in the lines exactly through the points obtained. The general form of the function $y(x)$ describing the phenomenon is most commonly known. For example, in a horizontal projection from y_0, the body height for different coordinates x is determined by the function:

$$y = y_0 - \frac{g}{2v_0^2}x^2 \,, \tag{2.5}$$

The graph of the above dependence is a parabola, but its exact shape becomes indefinite until the parameters y_0, v_0 and g are known.

In practice, we usually know the shape of the theoretical curve, but we do not know the parameters of the function. Theoretical function can be adapted to the measurement data using the *least squares method.*

Let's denote the value of the theoretical function at x_i by $y(x_i)$, and the value measured at this point by y_i. The deviation of the measured value from the theoretical value for each measurement is

$$y(x_i) - y_i \ , \tag{2.6}$$

Gauss's postulate says that the fit is best when the sum of the squares of deviations at all points takes the smallest value, which can be expressed in the form of the equation:

$$\sum_{i-1}^{n} [y(x_i) - y_i]^2 = \text{minimum} \ . \tag{2.7}$$

The use of the least squares method is easy only for linear functions - in this case the method is called *linear regression.* Condition (2.7) applied to the linear function

$$y = a_{reg}x + b_{reg} \ . \tag{2.8}$$

allows, after appropriate transformations, to find parameters characterizing a given straight line: *the slope factor* (a_{reg}) and *the intersection point* of the straight line with the y (b_{reg}) axis. These parameters can be calculated from the following equations:

$$a_{reg} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \ . \tag{2.9}$$

$$b_{reg} = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^2 - (\sum x_i)^2} \ . \tag{2.10}$$

where n is the total number of pairs (x, y), and the values of the indicator and range are $[1...n]$. Knowing the parameters a and b, you can draw the right straight line and find the value of y for any x based on the equation (2.8). When using linear regression, pay attention to whether the measuring points are randomly distributed relative to a straight line, because another distribution of points indicates that the function $y(x)$ is not linear.

EXAMPLE

Determination of straight line parameters based on the following measurements: The corresponding sums in equations (2.9) and (2.10) are:

x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.0	2.0	2.2	2.5	4.0	5.0	4.5	5.5

$\sum x = 18$, $\sum y = 26.7$, $\sum xy = 73.55$, $(\sum x)^2 = 324$, $\sum x^2 = 51$.

The parameters of the straight line have the values: $a = 1.28$, $b = 0.45$.

Linear regression can be used not only for explicitly linearly dependent quantities, but also for each pair of quantities described by a function that can be represented in a linear form by the use of appropriate substitutions (see formula 2.2 followed by the description).

An important advantage of using a linear regression to increase the accuracy of the parameters calculated in this way in relation to the values obtained directly from the graph.

Computer calculations

Most often they are repeated calculations, the arithmetic mean, standard deviation and linear regression. Manually their performance or even using a calculator requires a lot of time, and does not teach new skills. To perform these calculations quickly, take advantage of a computer program *StatS.exe* now named as *Measurement statistics* in PL and EN language version, specially designed for physics lab. The program is an addition to this script and is available on the website http://www.phys.put.poznan.pl. To get the required results, you should only enter data into the table, and the corresponding calculations will be done automatically.

In the case of single-size measurements, only one column is filled in, and the results window contains the arithmetic mean, standard deviations and variances (Fig. 2.3). The basic measure of error of the mean is the standard deviation of the mean, and other expressions used in appropriate cases. The drawing also contains a *histogram*, i.e. a bar graph showing the number of measurements obtained (vertical axis) in a series of value ranges (horizontal axis). The histogram illustrates the distribution of measurements, which should take the shape of a Gaussian curve for a sufficiently large series of measurements.

Figure 2.4 shows the data window of the pair of sizes and the window of the sizes characterizing the linear regression - slope factor, intersection with the axis Y, errors (uncertainties) of both these quantities, and also the correlation coefficient. The figure also has a graph in the form of points (x, y) and a straight line with parameters calculated from linear regression. This graph also provides visual information if the measurement points are randomly distributed relative to a straight line (only if regression can be used).

A very useful function of the program is *data conversion*, which consists in performing operations on all data of the selected column. For example, for the X column, we calculate the inverse $(1/x)$, and for the Y column, logarithms $(\ln y)$.

The charts in the appropriate boxes are only a simplified illustration of the results and should not be used in this form to present the results developed.

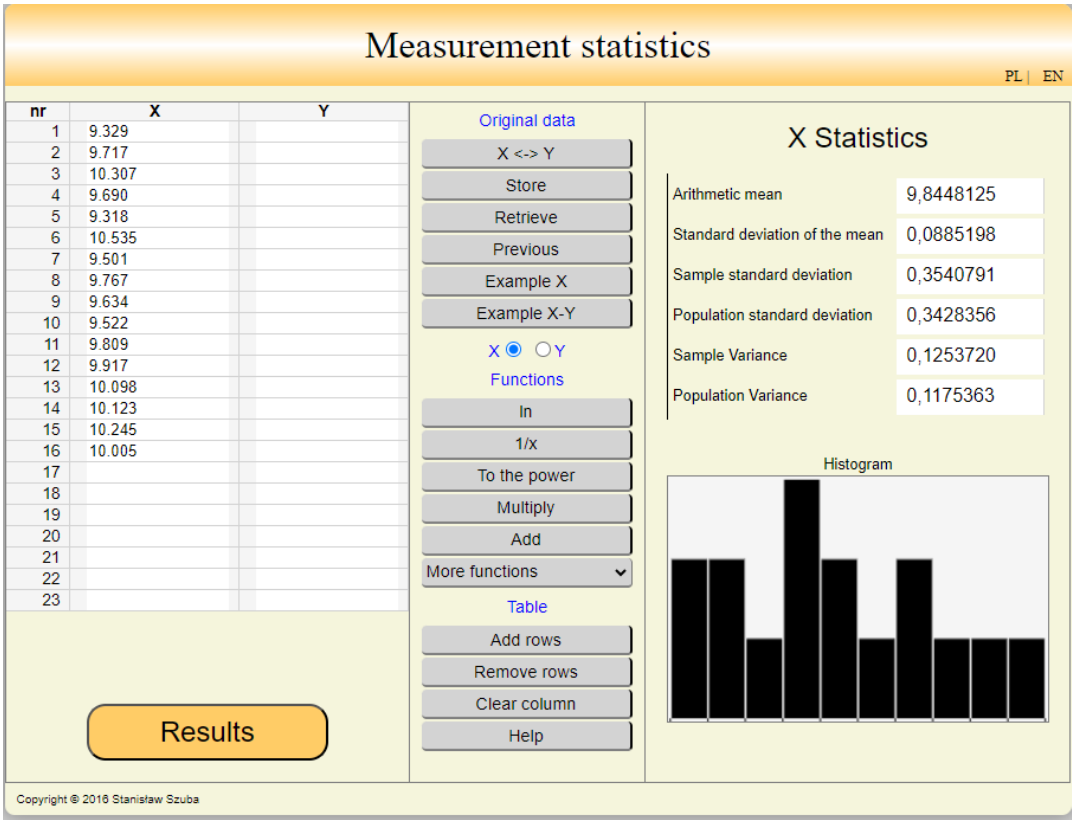


Figure 2.3. “Measurement statistics” program - data and results pane for one physical quantity in x column

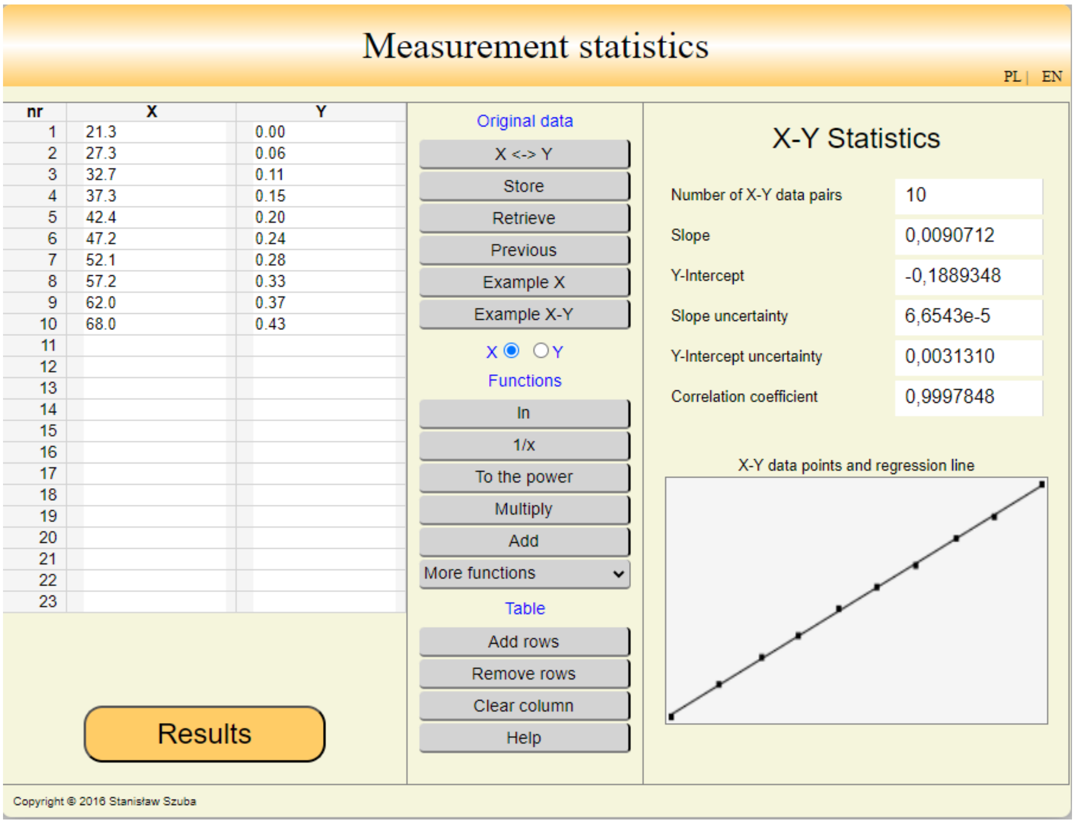


Figure 2.4. "Measurement statistics" program - data boxes and results regarding a pair of physical quantities

3. Error evaluation

Sources of Errors

The measurements taken in the laboratory never correspond exactly to the actual value of the measured value. They are subject to a greater or lesser error. *The measurement error* is the difference between the measured value and the actual value. *The true value* may be smaller or larger than the measured value.

If we denote the measured value by x, the true value by x_0, and the reading accuracy by Δx, then on the basis of the above remarks we can say that the true value lies in the range between $x - \Delta x$ and $x + \Delta x$. The measurement result should be written as $x_0 = x \pm \Delta x$.

EXAMPLE

A thermometer having 1 °C graduations indicates $T = 24$ °C. The measurement error is 1 °C. The actual temperature is in the range (23 ÷ 25 °C). Temperature measurement result T = (24 ± 1) °C.

Systematic errors

Systematic errors result from the inaccuracy of instruments, from the use of the wrong measurement method or from external factors. Measuring instruments are constructed in such a way that the results of correctly carried out measurements do not differ from the actual value more than by the value of the smallest division of the scale, which is why we call *reading accuracy.*

In the case of electric switches, accuracy may be determined by *the class of the instrument*, i.e. a number indicating the ratio of the systematic error to the meter range. For example, a class 0.5 meter with a 100-scale scale is half the scale smaller.

The development of a proper *measurement method* is not easy from taking into account the various phenomena accompanying the measured phenomenon, as well as the impact of measurement on the measured quantity. For example, measuring the run time of competitors at 100 m by the judges at the finish and starting the stoppers aloud with the starter's shot gives a lower value than the actual time needed for the acoustic wave to reach the judges. Not taking this delay into account causes a systematic error of about 0.3 s

Using the wrong method of measurement may cause a change in the measured value. For example, by inserting a mercury thermometer into a small vessel with liquid, we will not measure the proper liquid temperature, but the temperature determined as a result of heat exchange between the thermometer and the liquid. In this case, a proper measuring method would be to use a thermocouple, which practically does not take heat from the tested body due to the low value of its own heat capacity.

Incorrect measurement method also includes observing the instruments indications from the wrong angle, which leads to the so-called *parallax error*, and the use of approximate formulas to calculate complex quantities.

Measurement method errors are most often difficult to quantify. In practice, the systematic error is usually the accuracy of the instrument.

Random Errors

Repeating the measurement of a simple size many times with a high accuracy device (small systematic error), we get a different result each time, and the differences between the readings far outweigh the systematic error. The measured values are subject to random errors, whose source is the properties of the tested object (phenomenon), device or the person conducting the measurement.

By measuring, for example, the diameter of a wire with a micrometer, we obtain different results due to the change in diameter in different places and due to the different screw pressure. A special role is played by accidental errors in *subjective measurements*, such as comparing lighting, assessing maximum sound intensity or power loss. Accidental errors cannot be lost, you can reduce their impact on the final result and calculate their value. The methods for calculating random errors are described below.

Fatal errors

They are the result of mistakes that occurred during measurements (e.g. reading in centimeters instead of millimeters Erroneous errors exceed the remaining errors several times, so it is easy to notice them. The result burdened with a fat error is repeated or rejected.

Recognizing the type of error and repeating measurements

When starting the measurements, first of all we need to find out what kind of error occurs when measuring each of the simple quantities. In order to determine the type of error, *a test series* should be made: measuring the value three times. The results of this series will allow us to determine the type of error as well as the further way of measurement.

If all measurements in the trial series have the same value, we conclude that the measurement is dominated by systematic error. We take any of the measurements as the result, and the accuracy of the instrument as the error.

If the measurements in the trial series are different, a random error dominates the measurement. Then, increase the measurement series to at least 10, take the arithmetic mean as the measurement result, and the standard deviation of the arithmetic mean as the error (see formula 3.9).

Basic concepts of random error theory

Real value, arithmetic average

According to Gaussian theory, the measurement result should be taken as the value for which the sum of squares of deviations of individual measurements is the smallest. This value is the arithmetic average. If we make n measurements and denote the result of each of them by x_i ($i = 1, 2, 3 \ldots n$), we define *the arithmetic mean* as follows:

$$x_i = \frac{1}{n}\sum_{i=1}^{n} x_i \, . \tag{3.1}$$

Theoretical considerations show that for a sufficiently large number of measurements, the value of the arithmetic mean does not differ much from the actual value, therefore the error of the individual measurement is the deviation of the measurement value from the arithmetic mean

$$\varepsilon_i = x_i - x_s \, . \tag{3.2}$$

Error distribution

Analyzing the results of measurements of the same magnitude, we come to the conclusion that their greatest concentration is close to the average value. The number of results deviating from the mean decreases as the deviation increases. *The distribution of errors* is subject to statistical laws - it is described by *the Gauss function* expressed by equation (3.3) and presented graphically in Fig. 3.1.

$$\varphi(x - x_s) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-x_s)^2}{2\sigma^2}} \, . \tag{3.3}$$

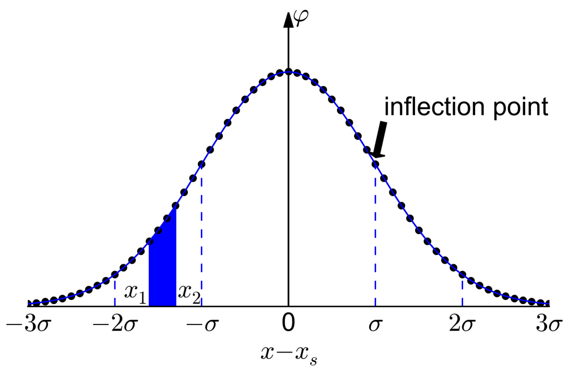


Figure 3.1. Gaussian curve of random error distribution

In the above formula, $\varphi(x - x_s)$ means the probability density of the measurement of the value x or an error of $x - x_s$, σ is a constant characterizing the measurement accuracy. The probability of obtaining the result, the range (x_1, x_2) is determined by the field between the x axis and the $\varphi(x - x_s)$ curve and the abscissa x_1 and x_2 (Fig. 3.1). This field can be calculated by integrating the distribution curves:

$$P = \int_{x_1}^{x_2} \varphi(x - x_s)dx \, . \tag{3.4}$$

The calculation of the integral from $-\sigma$ to $+\sigma$ leads to the conclusion that

68.3% of all errors fall within this range. The value of σ, which is abscissa of the Gaussian curve inflection point, is called *measurement standard deviation.*

Similarly, we can calculate that errors less than $|2\sigma|$ and $|3\sigma|$ are encumbered with 95.4% and 99.7% of all measurements, respectively. From the presented values it follows that in practice the maximum error of measurement should be equal to the tripled value of the standard deviation.

Calculation of errors of any measurement

The standard deviation of any measurement σ with a series consisting of a large number of measurements of equal accuracy is calculated using the formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i-1}^{n}\varepsilon_i^2}\,. \tag{3.5}$$

where n is the number of measurements and ε - deviation of the individual pore value from the arithmetic mean. The true value lies in the range $(x_i - \sigma,\ x_i + \sigma\ )$ where x is the value of any measurement, For numerical calculations, it is convenient to use the above formula transformed into the form:

$$\sigma = \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}}\,. \tag{3.6}$$

Calculators from different companies have an internal program, activated by the appropriate button, which allows to calculate the standard deviation after entering all the data x_j This button is most often marked with σ_{n-1} or S_{n-1}. *The average error* is defined as the arithmetic mean of the absolute values of all individual deviations.

The average error is defined as the arithmetic mean of the absolute values of all individual deviations

$$\varepsilon_p = \frac{1}{n}\sum_{i-1}^{n}|\varepsilon_i|\,. \tag{3.7}$$

The measurement error of a large series is smaller than the standard deviation; in theory it can be shown that

$$\sigma_p = 0.8\sigma\,. \tag{3.8}$$

Calculation of arithmetic mean errors

The arithmetic mean of many measurements of a given quantity does not coincide completely with the real value, however the range around the mean value in which we expect to find the real value is much smaller than the error of a single measurement. *The standard deviation of the arithmetic mean* is given by the formula:

$$\sigma_s = \sqrt{\frac{1}{n(n-1)}\sum_{i-1}^{n}\varepsilon_i^2}\,. \tag{3.9}$$

The true value lies in the range $(x_s - \sigma_s,\ x_s + \sigma_s)$, where x is the arithmetic mean of the measurements. The interpretation of both types of deviations σ and σ_s is shown on the numerical axis in Figure 3.2.

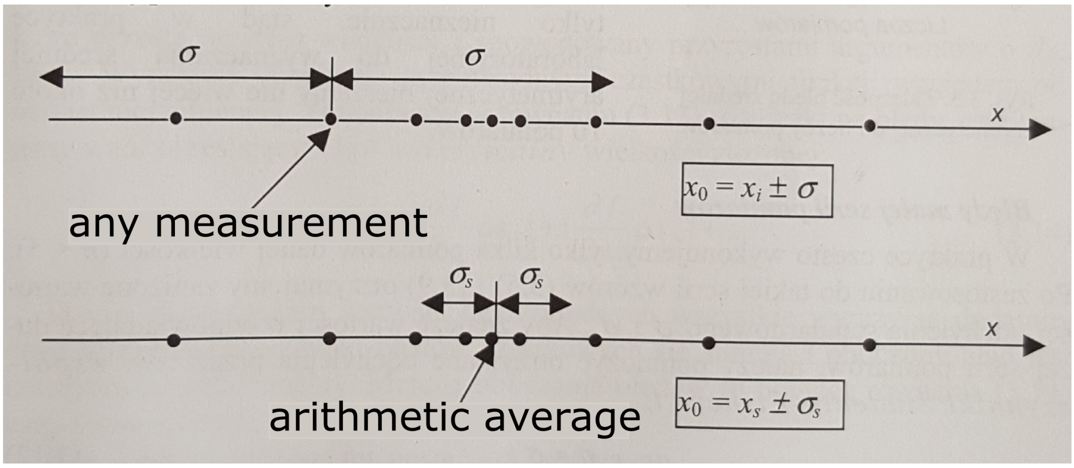


Figure 3.2. Illustration of the standard deviation of any measurement (σ) (above) and the standard deviation of the arithmetic mean (σ_s) (below) on the number line; black points indicate the value of measurements, x_0, x_s, x_i - true value, average, any measurement.

Comparing expressions (3.9) and (3.5), we conclude that the relationship:

$$\sigma_s = \frac{\sigma}{\sqrt{n}}\,. \tag{3.10}$$

The use of the above relationship is particularly common, we use a calculator with the σ calculation program.

We calculate *the average error of the arithmetic mean* on the basis of the expression:

$$\varepsilon_{ps} = \frac{1}{n\sqrt{n}}\sum_{i-1}^{n}|\varepsilon_i|\,. \tag{3.11}$$

The calculation of averages as well as standard deviations is quite labor-intensive, which is why it is worth using the *StatS.exe* computer program (see chapter 2) to facilitate these and other calculations.

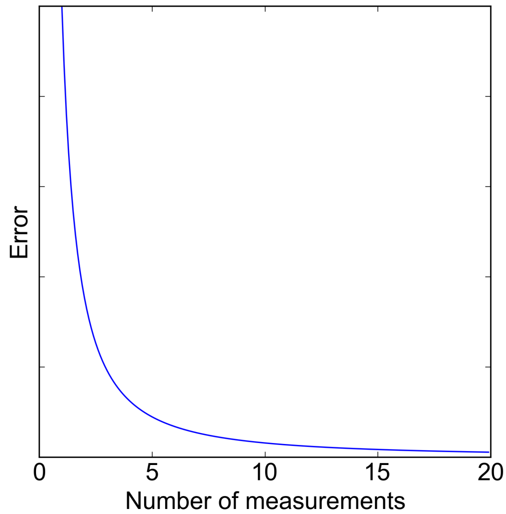


Figure 3.3. Dependence of the arithmetic mean error on the number of measurements

Comparing formulas (3.9) and (3.11) with formulas (3.5) and (3.7), we see that the arithmetic mean errors are $n^{1/2}$ times smaller than the errors of individual measurements and decrease with the increase in the number of measurements. The dependence of the error on the number of measurements is illustrated in the graph in Figure 3.3. As the graph shows, the number of measurements has a decisive impact on the error of the arithmetic mean in the range of small values ($n < 10$). Increasing the number of measurements after above 10-15 improves the accuracy of the result Number of measurements only slightly, hence in laboratory practice for determining the arithmetic average we take no more than 10 measurements.

Errors of small series of measurements

In practice, we often make only a few measurements of a given size (n <5). After applying to such a series of formulas (3.5) and (3.9) we get understated values of the standard deviation σ' and σ'_s. To find the σ values corresponding to a large series of measurements, multiply the deviations obtained by the so-called *Student-Fisher coefficients* t_n

$$\sigma = \sigma' t_n \, . \tag{3.12}$$

The coefficient values depend on the number of measurements n and the type of error being calculated. Table 3.1 contains the values of the most-used Student-Fisher coefficients for the number of measurements from 2 to 10 and for the standard deviation with a confidence level of 68.3 %.

Table 2.2. Student-Fisher coefficients for the standard deviation of the arithmetic mean (confidence level = 0.68)

n	2	3	4	5	6	7	8	9	10
t_n	1.84	1.32	1.20	1.14	1.11	1.09	1.08	1.07	1.06

Complex measurement errors

The errors discussed so far concerned measurements of quantities measured directly, such as: length, temperature or current. However, in laboratory practice, we most often determine a certain complex quantity based on the measurement of several simple quantities. The error of each of the measured quantities makes some contribution to the error of the result, with the errors of individual simple quantities can partially compensate or add up. Since we cannot assess the sign of errors, we choose the least favorable case, i.e. one in which each partial error increases the error of the result.

The calculation of the composite quantity error is based on a differential calculus. We treat measured quantities as function arguments, and their errors as differentials. Of course, this approach is justified when the errors are much smaller than the value itself.

Suppose that the value z is a function of the variables x_1, x_2, ... that is, $z = f(x_1, x_2, ...)$. The complete differential of this function is the expression:

$$dz = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + ... \, . \tag{3.13}$$

which determines the increase in size due to the increase of arguments by dx_1, dx_2, ... Fractional expressions are partial derivatives of a function relative to the corresponding variable. Changing the difference to equation in equation (3.13), we get the formula defining *the maximum error* of the complex value:

$$\Delta z = |\frac{\partial f}{\partial x_1}\Delta x_1| + |\frac{\partial f}{\partial x_2}\Delta x_2| + ... \, . \tag{3.14}$$

By using absolute values all words are positive, i.e. all contributions of partial errors add up and the calculated error is really maximum. The method of calculating errors using equation (3.14) is called *full differential method.* When the composite quantity has a product form, e.g.

$$z = cx_1^m x_2^n ... \, . \tag{3.15}$$

where c, m, n are the constants. It is more convenient to use *the logarithmic differential method.* Logging the equation (3.15) leads to the form:

$$\ln z = \ln c + m \ln x_1 + n \ln x_2 ... \, , \tag{3.16}$$

for which the total differential and then the relative error are calculated very simply:

$$\Delta z = (|0| + |m\frac{\Delta x_1}{x_1}| + |n\frac{\Delta x_2}{x_2}| + ...)z \, , \tag{3.17}$$

Equation (3.17) contains only measurement values and their errors, and does not contain derivatives, so using this method is more convenient than the methods of full differential. Therefore, before calculating the composite error, it is worth checking whether the function is a product.

As *direct measurement error* for equations (3.14) or (3.17) we take either systematic error (instrument accuracy) or random error (standard deviation of the mean), depending on which of them was recognized as dominant.

EXAMPLE 1

We determine the moment of inertia of the cylinder with mass m and radius R with respect to the axis parallel to the axis of the cylinder and distant from it by d. The results of direct measurements and their errors are as follows: $m = (55.3 \pm 0.1)$ g, $R = (3,52 \pm 0.01)$ cm, $d = (20 \pm 0.1)$ cm. We calculate the moment of inertia using Steiner's law

$$I = m(\frac{R^2}{2} + d^2)\,, \tag{3.18}$$

The equation has no product form, so we must use the formula (3.14). First, we calculate the appropriate partial derivatives:

$$\frac{\partial I}{\partial m} = \frac{R^2}{2} + d^2\,; \frac{\partial I}{\partial R} = mR\,; \frac{\partial I}{\partial d} = 2md\,. \tag{3.19}$$

then insert them into the formula (3.14) and obtain

$$\Delta I = |(\frac{R^2}{2} + d^2)\Delta m| + |mR\Delta R| + |2md\Delta d)|\,. \tag{3.20}$$

Numeric value of error $\Delta I = 263.7$ g/cm^2, and the measurement result should be saved in the final form $I = (22460 \pm 260)$ g/cm^2.

EXAMPLE 2

We determine the Young's modulus by means of bar deflection based on the relationship

$$E = \frac{l^3}{12\pi S r^4} F\,. \tag{3.21}$$

The quantities measured directly are i, S, r and F. The initial equation has a product form, so we will use the logarithmic differential method, i.e. the formula (3.17) adapted to the current quantities:

$$\Delta E = \left(3\frac{\Delta l}{l} + \frac{\Delta S}{S} + 4\frac{\Delta r}{r} + \frac{\Delta F}{F}\right)E\,. \tag{3.22}$$

Practical notes on calculating errors

1. We calculate errors of measured quantities directly according to the recognition type of error (see page 10 and section 3).
2. When calculating the composite quantity measured once, the maximum error we calculate by the total or logarithmic differential method.
3. We also use errors calculated according to point 1 as a direct measurement error average in expressions for the composite error (Δx in equations 3.14 and 3.17).

4. If the composite quantity is measured many times, then for the measurement result we take the arithmetic mean of these measurements.
5. Before repeating the measurements ($n > 2$) again, the composite quantity should be calculate the maximum error of this quantity according to point 2 and check whether the difference between the two measurements is greater than the maximum error. If it is not, discontinue further measurements and take the maximum error as the measure of the complex quantity error. If the difference is larger, it is advisable to increase the number of measurements, take the arithmetic mean as the result and the deviation as the error standard average.
6. When a series consists of a small number of measurements, we must take into account Student-Fisher coefficients.

4. Presentation of results

We have full information about the determined physical quantity when we know its value and the error it is burdened with. We must remember that the methods presented above do not allow for a precise determination of the deviation of the measurement result from the actual value. For example, the standard deviation specifies the range around the mean of the measurement, and the probability of finding the actual value in this range is 68 %.

Therefore, giving the result as well as an error in the form of a multi-digit number is pointless - the first two significant digits have a physical sense. The calculated error and result values must be *rounded*. The error value usually has two significant digits[1], sometimes one, never more. In the conditions of the physical lab we will use *rounding procedure* shown in Table 4.1.

When rounding the result, we use general rules, i.e. numbers from 1 to 4 are rounded down, and numbers from 5 to 9 are rounded up. Table 4.2 gives some examples of rounding results and errors,

The absolute error is the difference between the true value and the value obtained as a result of measurements.

The ratio of absolute error to (average) measurement value is called *relative error*

$$\varepsilon_w = \frac{x - x_{avg}}{x_{avg}} \, . \tag{4.1}$$

The final result must contain the rounded values of the measured quantity and error and units. As the culmination of the work, it should be presented in enhanced form, e.g. in capital letters, in a frame or in another color, e.g.

$$q = (69.37 \pm 0.02) \cdot 10^{-9} \text{ C}$$

We often express a relative error in percentage. To calculate the percentage error, it is sufficient to multiply the expression (4.1) by 100%. Providing a relative error allows

[1] Significant digits are obtained by rejecting the leading and trailing zeros. The position of the decimal point does not matter. Not to be confused with decimal digits!

Table 2.3. Procedure for rounding off errors and results

Action	Example 1	Example 2
1. We calculate the error value with high accuracy	$\Delta x_1 = 1932$	$\Delta x_1 = 0.05186$
2. We round the error to two significant digits	$\Delta x_1 = 1900$	$\Delta x_1 = 0.052$
3. We make a trial rounding of an error up to one significant digit	$\Delta x_2 = 2000$	$\Delta x_1 = 0.06$
4. Has Stage 3 resulted in a value change greater than 10%; we're checking inequality $(\Delta x_2 - \Delta x_1)/\Delta x_1 > 0.1$	no	yes
5. Yes - we leave 2 significant digits. No - we leave 1 significant digit.	$\Delta x_2 = 2000$	$\Delta x_1 = 0.052$
6. We calculate the measurement result by at least one place decimal further than the place where the error was rounded	x=26231	x =0.3794
7. We round to the decimal place to which the error was determined	x=26000	x =0.379
8. Final record	x=26000±2000	x=0.379±0.052

Table 2.4. Examples of rounding off errors and results

Before rounding	After rounding
$r = (225.173 \pm 0.191)$ cm	$r = (225.2 \pm 0.2)$ cm
$t = (7.5752 \pm 0.0234)$ s	$t = (7.575 \pm 0.023)$ s
$i = (93.311 \pm 0.092) \cdot 10^{-3}$ A	$i = (93.3 \pm 0.1) \cdot 10^{-3}$ A
$C = (0.2266 \pm 0.00282)$ μF	$C = (0.227 \pm 0.003)$ μF
$q = (69.4659 \pm 0.0357) \cdot 10^{-9}$ C	$q = (69.466 \pm 0.036) \cdot 10^{-9}$ C
$q = (69.3659 \pm 0.0187) \cdot 10^{-9}$ C	$q = (69.37 \pm 0.02) \cdot 10^{-9}$ C
$G = (4567893 \pm 32331)$ N	$G = (4568000 \pm 33000)$ N

you to quickly evaluate the result. In the conditions of the physical laboratory, we consider satisfactory results with a relative error $\varepsilon_w < 0.1$.

A full graphical representation of the result must include errors in both quantities forming a functional relationship. The error of the measuring point on the chart is marked by surrounding it an error rectangle (Figure 4.1) whose sides are equal to twice the value of the coordinate error. In principle, the curve should be followed after applying errors. If the measurements are carried out correctly, the smooth curve passes through at least 70% of

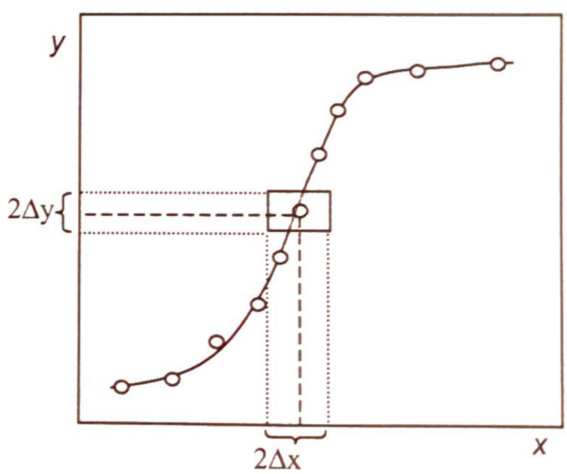


Figure 4.1. Error rectangle design

the error rectangles and the number of measuring points lying on both sides of the curve is approximately the same.

It is often necessary to compare two results obtained by different methods or / and compare with the value given in the tables. We denote such results and their maximum errors by:

$$A_1 \pm \Delta A_1 \text{ and } A_2 \pm \Delta A_2 \, . \tag{4.2}$$

From the property of the maximum error it follows that the real value of Ao should be simultaneously in two ranges

$$< A_1 - \Delta A_1 >, < (A_1 + \Delta A_1 > \ . \tag{4.3}$$

$$< A_2 - \Delta A_2 >, < (A_2 + \Delta A_2 > \ . \tag{4.4}$$

We consider the results to be consistent when both compartments partially overlap or are at least tangent. The condition of interruption of the intervals is offset by the following inequality

$$|A_1 - A_2| \leq |\Delta A_1| + |\Delta A_2| \, . \tag{4.5}$$

If A_1, is the result of experiment, and A_2 the value from the tables, then most often the error ΔA_2 is much smaller than ΔA_1 and approximately $\Delta A_2 = 0$. In these conditions the measurement result should satisfy the inequality:

$$|A_1 - A_2| \leq |\Delta A_1| \, . \tag{4.6}$$

If the inequalities (4.5) or (4.6) are met, then we say that the compared results are consistent.

Other reference materials: H. Szydłowski, Pracownia fizyczna, Warszawa, PWN 2003 [7].

3. Basic measuring instruments

5. Length measuring instruments

Caliper

The caliper is a device used to measure small lengths - up to several centimeters - with an accuracy of 0.1 or 0.05 mm. It consists essentially of a metal bar C (Fig. 5.1) ended with a jaw A and a movable slide D. The fixed part is marked with a millimeter scale, and on the slider there is a vernier with 10 or 20 divisions. To measure the external dimension of a body, place it between the A and B jaws and press them lightly. To measure internal dimensions, we use jaws E and F, which we insert into the measured hole and spread it apart until the stop. In order to measure the depth of the hole, rest the bar C on the upper edge, and insert the end of the slider G until it rests on the base of the hole.

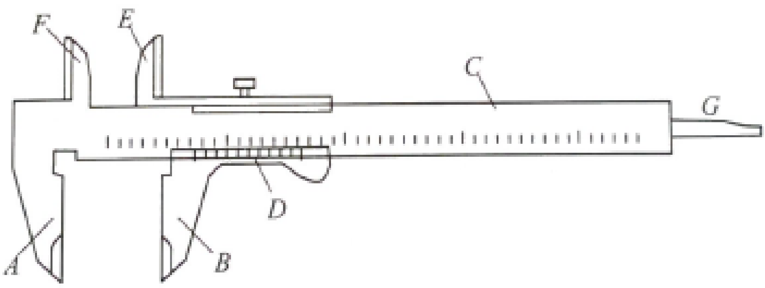


Figure 5.1. Vernier caliper; A, B - jaws for measuring external dimensions, C - fixed part with the main scale, D - sliding slider with a vernier scale, E, F - jaws for measuring internal dimensions, G - depth measuring tip

The principle of reading the vernier is shown in Fig. 5.2, which shows a fragment of the main scale and the entire vernier scale consisting generally of N divisions (in Figure N = 5). The reference point for the scale is the zero division of the vernier. The value of the smallest division of the d_n vernier is selected in relation to the smallest division of the d_s scale that the entire length of the vernier consisting of N divisions includes $(N-1)$ of the scale divisions, i.e.

$$Nd_n = (N-1)d_s, \tag{5.1}$$

From the above equation we find that the difference in the value of the plots ($d_s - d_n = \Delta$

$$\Delta = \frac{d_s}{N}. \tag{5.2}$$

This quantity is the *accuracy of reading* with a vernier scale. In any measurement, the zero vernier line is between the scale marks. At the same time, one of the vernier lines coincides with a certain line on the scale.

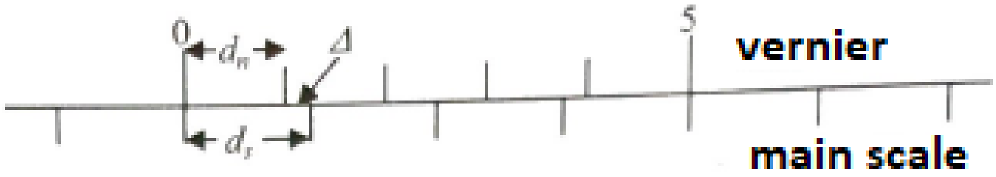


Figure 5.2. Vernier caliper; A, B - jaws for measuring external dimensions, C - fixed part with the main scale, D - sliding slider with a vernier scale, E, F - jaws for measuring internal dimensions, G - depth measuring tip

There are two steps to reading with a vernier:

- we find the value of the dash on the scale closest (but towards the smaller ones value) of a zero vernier line,
- we add the value of the product of accuracy and the number of the vernier line extending the scale.

For the caliper in fig. 5.1: $d_s = 1$ mm, $N = 10$, $\Delta = 0.1$ mm, the measured value is 7.3 mm. Vernier is used to measure distances and angles. In practice, vernier with the parameters presented in Table 3.1 is most often used.

Table 3.1. Parameters of the most commonly used vernier

d_s	N	Δ
1 mm	10	0.1 mm
1 mm	20	0.05 mm
30’	30	1’
15’	30	30”

Micrometer

When it is necessary to measure with an accuracy of 0.01 mm, then we use a micrometer, formerly known as a micrometric screw. The micrometer (Fig. 53) consists of the C stationary part with the S_1 scale, and the W drum with the S_2 scale. On the upper part of the S scale, millimeters are marked, and the lower scale divisions are halfway between the upper divisions. Typically, one full revolution of the bobbin case corresponds to 0.5 mm of the B tip; the S_2 scale has a range from 0 to 50.

The reading consists in adding the indication on the drum to the largest value of the uncovered scale of the fixed scale S_1. For example, if the bottom bar to the right of the 7mm division is visible and the bobbin gauge is 35, the reading would be 7.85mm.

Excessive pressure of the tip of the screw on the measuring object may deform it or damage the thread of the screw. In both cases the measurement will be false. In order to ensure constant pressure, the bolt is turned only by the D knob, which is connected to the other part through a clutch that disables the rotation of the bolt with excessive pressure.

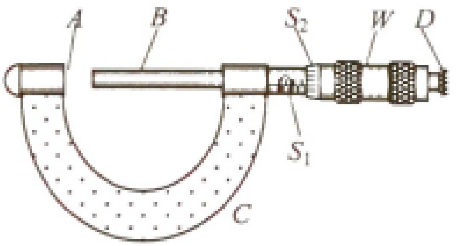


Figure 5.3. Micrometer, A, B - tips between which the measured object is placed, C - handle, D clutch knob, S_1 - fixed scale, S_2 - movable scale, W - rotated cylinder

Figure 5.4. Micrometric sensor

Before starting to measure, check the zero indication after bringing the A and B terminals into contact. If the micrometer does not show zero then read the shift of the zero reading from the beginning of the scale. The value of this shift must be included in the measurements, adding it to the reading or subtracting it from it, depending on the direction of the shift.

Micrometric sensor

The micrometer sensor measures length changes with an accuracy of 0.01 mm. The end of the slider (Fig. 5.4) is pressed by a delicate spring against the surface of the measured object. The displacement of the surface is transferred through the slider and the system of gears to two pointers, the larger of which performs one revolution when the slider is moved by 1 mm, and the smaller - when the slider is moved by 10 mm. The movable scale enables convenient setting of the zero position.

6. Laboratory scales

Construction and operation

Balance is a device used to determine body mass or weight. In *spring* and *torsion scales* use the elastic properties of bodies. In particular, the proportionality of deformation to the acting force. When the force is the weight of the body, then the strain (elongation or twist angle) is proportional to mass as well. Determining the mass is possible when we know the value of the acceleration due to gravity, which is not constant but changes with the distance from the center of the Earth. This disadvantage does not occur in beam scales.

The method of measurement with the use of a beam balance consists in comparing the tested mass with a reference mass in the form of weights. The measurement result is independent of the force of gravity.

The structure of the balance is shown in Fig. 6.1, where the main elements are marked: a beam, pans, pointer, supporting prisms and an arresting device. The beam and pan are supported by prisms made of very hard material (often agate), which minimizes the impact of friction and allows for precise definition of the arm length. The pointer, rigidly connected to the beam, defines its position relative to the horizontal. The condition for proper operation of the scale is horizontal positioning of the base, which is achieved by adjusting it with two front legs, and controlling it with a spirit level or plumb line.

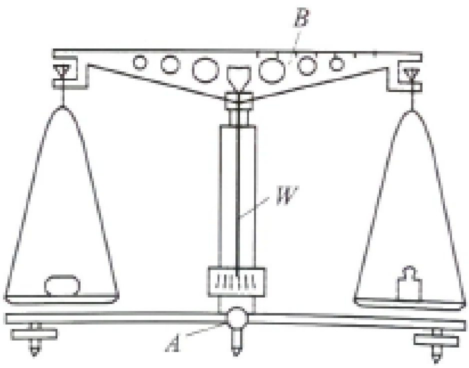


Figure 6.1. Laboratory scales; A - arrest mechanism, B - beam, W - hint

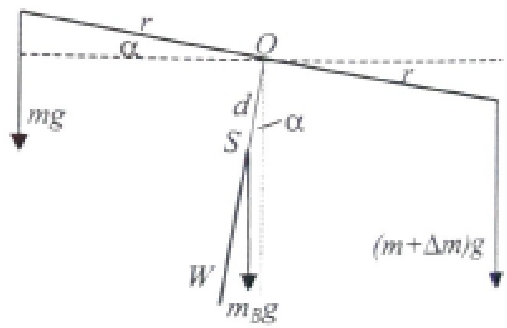


Figure 6.2. Forces in unbalanced weight

The utility value of a balance is determined by two parameters: accuracy and sensitivity. We take the value of the smallest weight as the *accuracy* of the balance, and *sensitivity* is the ratio of the pointer deflection angle to the excess weight which caused this deflection:

$$c = \frac{\alpha}{\Delta m}. \tag{6.1}$$

An equivalent term is often used in which the angle is replaced by the number of scale divisions - a

$$c = \frac{a}{\Delta m}. \tag{6.2}$$

Let us consider an isosceles balance whose arms are unequally loaded - one pan has the mass m, and the other one $m + \Delta m$ (Fig. 6.2). The beam tilts from its horizontal position by α. Note that the equilibrium state, despite unequal loads, can only exist when the support point of the O beam is above the center of gravity. For the system to be in equilibrium, the resultant *moment of force* about the point O must be zero. The moment of force acting on the right arm is $(m + \Delta m)gr\cos\alpha$, we take it as positive. The two remaining moments try to rotate the beam in the opposite direction, so they have a negative value: $-mgr\cos\alpha - m_B gd\sin\alpha$, where d is the distance of the axis of rotation from the center of gravity, and m_B - the mass of the beam. The equilibrium condition takes the form of the equation:

$$(m + \Delta m)gr\cos\alpha - mgr\cos\alpha - m_B gd\sin\alpha = 0. \tag{6.3}$$

Dividing the above equation by $\cos\alpha$ and making simple transformations, we find the tangent of the deflection angle

$$\tan\alpha = \frac{r\Delta m}{dm_B}. \tag{6.4}$$

When the deflections are small, we can assume that formula (6.4) also determines the deflection angle itself (because in this case $\tan\alpha \approx \alpha$) and use it to determine the sensitivity based on the dependence (6.2). By doing the substitution, we get:

$$c = \frac{r}{dm_B}. \tag{6.5}$$

The sensitivity of the balance is therefore proportional to the length of the arms and inversely proportional to the weight of the beam and the distance of the point of support from the center of gravity. In modern scales, fairly short beams made of light aluminum alloys are used. Thanks to this structure, the scales are convenient to use (small dimensions) and sufficiently sensitive (favorable ratio of the beam length to its weight).

If the beam was supported at the center of gravity ($d = 0$), then - as equation (6.5) shows - its sensitivity would be infinitely high. Such a weight would be useless as any difference in mass would cause the beam to tilt into position vertical!

Each scale can only be loaded up to a certain limit, above which the structural elements are deformed and the measurement results become erroneous.

Each scale includes a *set of weights* with values in the sequence: 1, 2, 2, 5. The smallest weight is usually 10 mg. *Analytical balances* placed in special housings and having air damping of fluctuations have greater accuracy. Their accuracy ranges from 0.1 mg to 0.01 mg. 0.01 mg.

Weighting

There are several rules that you must follow in order to use a scale and to obtain correct results.

All manipulations on the weighing pan, i.e. adding weighing scales, placing the weighed body, etc., should be performed on the scales. The principle of arresting comes down to removing the beam and trays from the prisms and supporting them on special immobilizing stands. The A knob is used for detecting and must be turned with a gentle movement.

The weighing cabinet should be opened only when necessary, thus avoiding the harmful effects of air currents and temperature changes. For applying weighing scales, it is enough to open the cabinet wall halfway. The application and removal of weights should be performed with the use of special forceps, which are the balance accessories.

We start the weighing process with leveling the balance, checking pans for cleanliness and setting the zero position of the pointer. When the center of fluctuation of the unloaded weight does not coincide with the center of the scale, then the center of gravity of one of the weighing panes is shifted by appropriate tightening of the horizontal screws at the ends or in the middle of the beam.

Place the tested body on the left pan and put the weights on the right pan. Then we find the smallest weight that causes overweight, then replace it with a half weight and add smaller weights until we get another overweight. Proceed in this way until the smallest weight from the set is used.

In order to achieve an accuracy greater than the value of the smallest weight, we use *the interpolation method* in which we take into account the position of the pointer. This method requires a precise *zero position*. Suppose that after reset, the pointer is skewed to the right p_1, to the left - l_1, and the second to the right - p_2. The center of fluctuation is calculated according to the formula:

$$S_0 = \frac{\frac{p_1+p_2}{2} + l_1}{2} = \frac{p_1 + p_2 + 2l_1}{4}. \tag{6.6}$$

If the weight scale has zero in the middle, then the left tilt is negative, and the right tilt is positive. In a similar way, we determine the center of S_- fluctuations with the smallest underweight. The difference $S_0 - S_-$ is proportional to the difference between the weight of the weights and the weight of the body

$$x \propto S_0 - S_-. \tag{6.7}$$

Based on equations (6.6) and (6.7), we find the addition to the mass of weights giving the result with an underflow:

$$x = \frac{S_0 - S_-}{S_+ - S_-}\Delta m. \tag{6.8}$$

As in formula (6.6), the S values have different signs, depending on their position in relation to the zero scale value. The value of x is rounded to one significant place. The final result of mass measurement using the interpolation method is calculated from the formula

$$m = m_- + \frac{S_0 - S_-}{S_+ - S_-}\Delta m. \tag{6.9}$$

where: m_- is the weight of the least underweight and Δm is the weight of the overweight result. The above method allows to determine the mass value with an accuracy that exceeds the mass of the smallest weight by one order.

In *automatic analytical balances*, weights are put on by turning an appropriate knob, and in *semi-automatic balances*, larger weights are placed manually, smaller weights - automatically. Locations determining values less than 10 mg are determined from the deflection of the pointer read with a precision optical system.

7. Ultrathermostat

In many physical measurements it is necessary to keep the body or medium at a temperature different from the ambient temperature. For automatic temperature control, we often use an ultrathermostat - a device in which heat is supplied to the system when its temperature is lower than required. The ultrathermostat contains the following (see figure 7.1):

- a tank with a thermostated liquid,
- a controlled heater, an additional heater,
- cooler,
- temperature sensor (contact thermometer, thermistor, diode, etc.),
- control system,
- mixing and forcing pump.

The liquid reservoir may be connected to an external device that is allowed to flow by a pump driven by an electric motor. The heater control system receives information about the current temperature from the sensor and turns on the heater when the temperature is lower than the required temperature, or turns the heater off when the temperature has reached the required value. The basic feature of the sensor must be a strong dependence of its properties (e.g. length, electrical resistance) on temperature.

One of the temperature sensors used is a *contact thermometer*, the structure of which is shown and the principle of operation is explained in Fig. 7. The lower part of the capillary serves as a thermometer, while the upper part is flared and houses the P, screw with the K nut on it, which in turn is connected to the D wire. When the screw is rotated, the nut makes a translational movement up or down, thus changing the position of the end of the wire relative to the mercury level. When the end of the wire touches the mercury, an electrical circuit is completed between terminals Z_1 and Z_2.

The upper end of the screw is provided with a small magnet that can be rotated with a larger external magnet. By turning the magnet, we set the cap to a certain temperature value, at which the mercury column is at the level of the wire end, which closes the circuit.

The Z_1, Z_2 terminals are connected to a relay that turns on the heater when the thermometer is open, and turns the heater off when the thermometer circuit closes. The heater is turned off and on at slightly different temperature values - this difference is called the *control insensitivity* and should be as small as possible. If you want to reach the required temperature faster, switch on the auxiliary heater. Since it does not respond to the contact thermometer, it must be switched off manually before the desired temperature is reached.

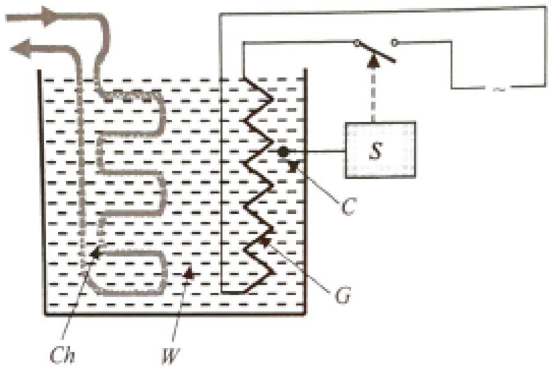


Figure 7.1. Thermostat diagram; S - control system, C - temperature sensor, G - heater, W - thermostated fluid, Ch - cooler

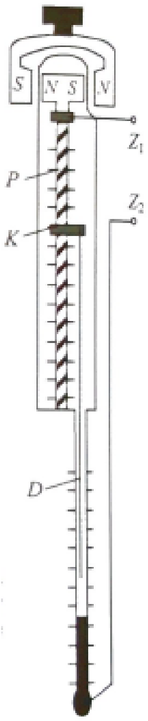


Figure 7.2. Contact thermometer; D - contact wire, K - nut moved along the threaded rod P through its rotation caused by manual rotation of the external magnets $N - S$, Z_1, Z_2 - terminals connecting with the heating circuit relay

In order to lower the temperature of the thermostating liquid, lower the cap (contact thermometer indicator) to the desired value - the heater is thus turned off - and we pass a stream of cold tap water through a spiral cooler immersed in the liquid. Of course, the lowest temperature that we can achieve in this way is the temperature of the cooling water. Correct control of the temperature slightly above the room temperature is achieved by simultaneously switching on the automatic heating system and circulation of cold tap water through the cooler.

8. Potentiometer and autotransformer

The basic power sources are: AC mains, galvanic cells and batteries. *AC voltage* varies with time according to the formula: $U = U_0 \sin(2\pi f t)$, where frequency $f = 50$ Hz and amplitude $U_0 = 324$ V. A more useful parameter is the RMS voltage U_s, which is smaller than the maximum by factor 1.41 and equals 230 V.

Galvanic cells and batteries are direct current sources. The electromotive force of a single cell of a lead battery is 2.2 V and the EMF of different cell types is from 1.0 V to 2.5 V.

In a physics laboratory, it is often necessary to regulate the voltage in a smooth or jumpy manner. The simplest device for this purpose is a *potentiometer*, a diagram of which is shown in Fig. 8.1. To the ends of the resistor R_0 we connect the voltage source U_0, as a result of which the current $i = U/R_0$ flows through the resistor. The drop in potential across the part of the resistor whose resistance is R is the product of the current and this resistance, so:

$$U = \frac{R}{R_0} U_0. \tag{8.1}$$

By changing the position of the slider (in the picture - contact ending with an arrow), we can get U voltage in the range from 0 to U_0.

Apart from potentiometers with a smooth slider movement, there are also potentiometers with step regulation; they are called *voltage dividers*.

Potentiometers can be used to adjust DC and AC voltage. Their disadvantage is that they draw current, sometimes much greater than the current of the actual receiver.

We often use an *autotransformer* to regulate AC voltage. The structure of this device is shown in Fig. 8.2. A single-layer winding is wound on the ring core, to the ends of which we connect an AC source. The reduced voltage is collected from the autotransformer by means of a slider in the same way as in a potentiometer. The winding resistance is an inductive resistance and only occurs for alternating current. Turning on the DC voltage would cause a large current to flow and burn the winding.

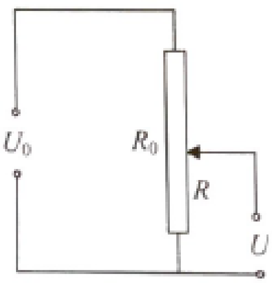


Figure 8.1. Potentiometer

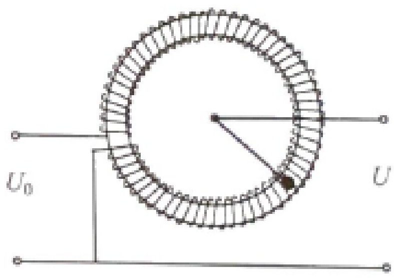


Figure 8.2. Autotransformer

9. Adjustable resistors

The slide resistors have a resistance wire winding wound around a ceramic core. The ends of the windings are brought to the terminals located on the outer side of the housing. Also outside there is a handle and a clamp for the slider, i.e. a contact sliding along the winding. When we connect the A and C terminals to the electric circuit (Fig. 9.1), the current will flow through the winding part between the A terminal and the slider, and then through the slider rail, the resistance of which is very small, to the C terminal. Of course, after switching on the B and C terminals, the winding resistance from the B point to the slider is "active".

Figure 9.1 also shows a schematic of a resistor having two windings, which we can use either separately or both together. Terminating both ends of the resistance winding with terminals allows the use of slide resistors as potentiometers.

A decade resistor consists of a plurality of wire resistors connected in such a way that any value can be selected, from the smallest single resistor to the sum of all. The

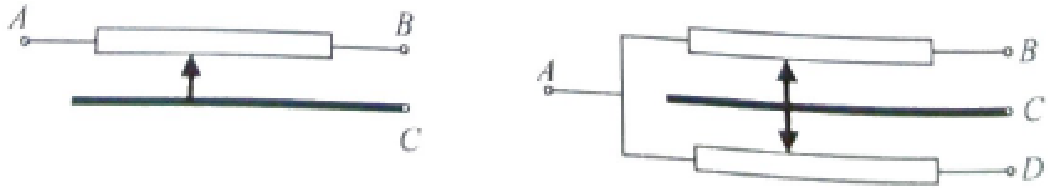


Figure 9.1. Slide resistors: single (left) and double (right)

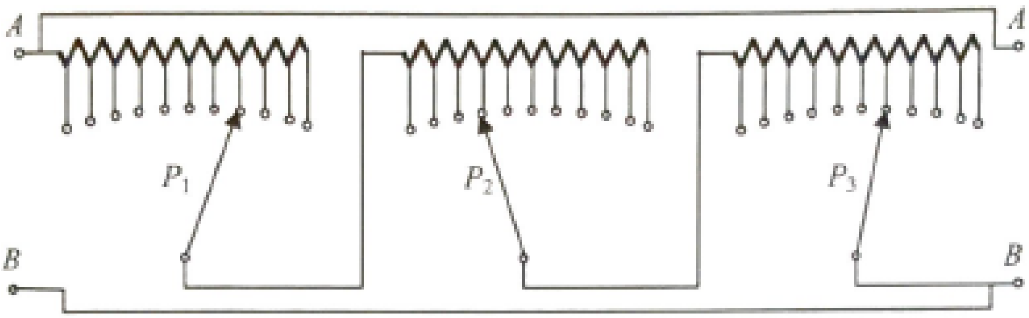


Figure 9.2. Decade resistor

values in the following decades differ by a factor of 10. The connection method in a 3-decade resistor is shown in Fig. 9.2. The resistance of a single decade is selected with the P switch, rotated by a knob with a value indicator from 0 to 9 (or 10 - in some constructions). Total resistance is the sum of all decades. The external circuit is connected to the A and B terminals at both ends of the resistor.

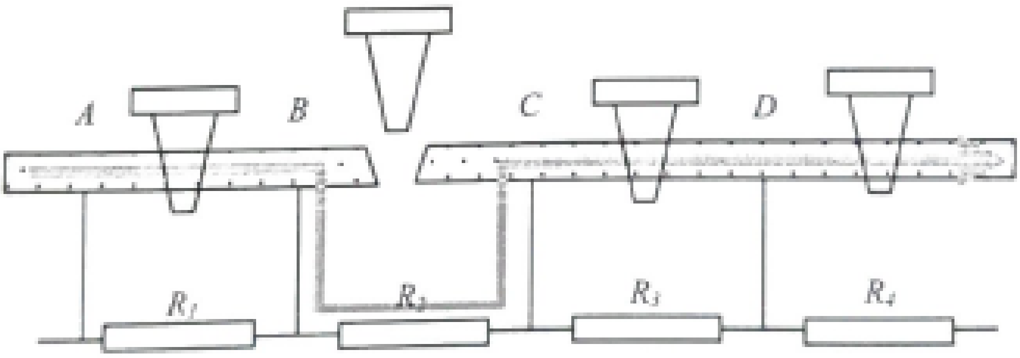


Figure 9.3. Plug resistor

A plug resistor is a system of series connected wire resistors, each of which can be disconnected from the circuit by shorting its ends with a special stopper/plug. (Fig. 9.2). All wire resistors are connected in series on a common copper bar, the resistance of which is very low. Between the ends of the individual resistors, the rail has gaps in the form of conical holes closed with matching metal plugs.

When the plug is in the hole, it connects a suitable resistor and the resistance of the section (eg. $A - B$ in Fig. 9.2) is equal to zero. When the plug is removed, the current must flow through a suitable resistor (eg R_2 in Fig. 9.2). The value of the resistance applied in this way is marked on the plate next to the opening. In order for the plugs to introduce additional resistance, they should be pressed in firmly. Resistors are generally sorted by decades. We calculate all the resistance turned on by summing up all the values corresponding to the removed plugs. There are auxiliary openings next to the rail break, which do not constitute a break in the electric circuit of the rail and are used to insert free plugs.

10. Switches

For the convenience of measurements, switches are used, with which we can easily close or open the circuit, change the direction of the current, and also change the components of the circuit.

The structure of typical 2-position switches is shown in Fig. 10.1. and their application to change the direction of the current - in Fig. 10.2.

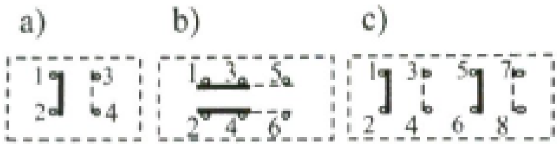


Figure 10.1. 2-position switches: a) 4-point, b) 6-point and c) 8-point; thick lines show internal connections of points for one item, dashed lines - for another

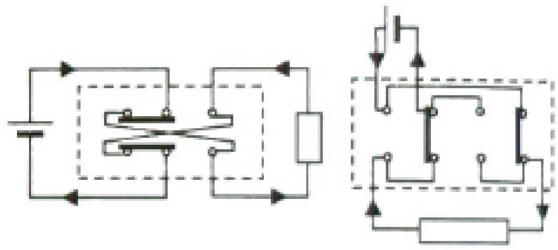


Figure 10.2. Change the direction of the current with switches

11. Electric meters

Magnetoelectric meters

The basic part of the meter (Fig. 11.1) is a frame, made of coils of thin copper wire, and a magnet. The frame is mounted on an axle in bearings and can rotate in the slot. In order to increase the magnetic induction, a stationary core is placed inside the frame, which further focuses the induction lines, so that the induction distribution in the slot is almost radial.

The measured current flows to the coil through spiral springs, in which, after twisting, a spring force moment (M_S) is generated, proportional to the coil angle φ. This property is expressed by the equation:

$$M_S = -k\varphi. \tag{11.1}$$

W polu magnetycznym o indukcji B na ramkę działa moment siły elektrodynamicznej

$$M_e = IBSn. \tag{11.2}$$

gdzie: I – natężenie prądu, S - powierzchnia przekroju ramki, n - liczba zwojów ramki.

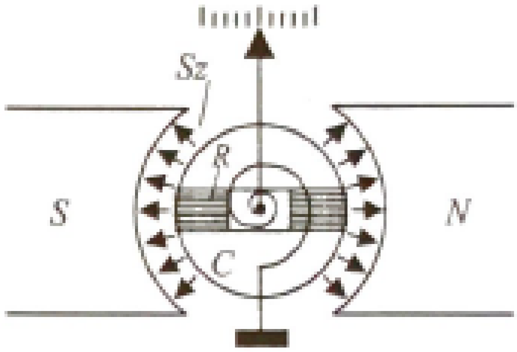


Figure 11.1. Basic elements of a magnetoelectric meter - N, S - poles of a permanent magnet, C - magnetic core (stationary), R - frame with wound winding

Both M_S and M_e act in the opposite way, which causes the frame tilt to become fixed when $M_S - M_e = 0$. From the momentum equilibrium condition it follows that the frame deflection angle is proportional to the current intensity, i.e.

$$I = A\varphi. \tag{11.3}$$

From equation (11.3) it follows that the scale of the magnetoelectric meter is linear.

Electrodynamic meters

The principle of construction of electrodynamic meters is similar to the principle of construction of magnetoelectric meters described above. The difference is replacing the permanent magnet with an electromagnet. The magnetic induction B produced by the electromagnet is proportional to the current strength I_1 flowing through its winding

$$B = k_1 I_1. \tag{11.4}$$

In this situation, the moment of the electrodynamic force is proportional to both the coil current I_2 and the electromagnet current:

$$M_e = k_1' I_1 I_2. \tag{11.5}$$

Similarly, the product of the two currents has a proportional deflection angle. When the same current flows through the electromagnet and the coil ($I_1 = I_2 = I$), then the deflection is not proportional to the square of the current intensity

$$\varphi \propto I^2. \tag{11.6}$$

Thus, the scale of the electrodynamic gauge is non-linear - the graduations become thinner as the deflection increases.

Ammeter range change

The meters described above are used to measure the current intensity. Due to the scope, they are divided into ammeters, milliammeters, microammeters and galvanometers. The latter are the most sensitive meters - they are used to measure currents of $10^{-6} - 10^{-11}$ A.

Placing the ammeter in the circuit should not change the value of the flowing current, so the resistance of the coils (internal resistance) should be very small.

For each meter there is a specific maximum current which, when exceeded, causes the emission of heat in the measuring coil, leading to winding damage. The measuring range of the meter is related to the maximum current.

The measured current may exceed the maximum value many times (let's denote it by I_m), if a suitable resistor is connected parallel to the terminals of the ammeter, the so-called *shunt* (fig. 11.2). Let's calculate what shunt we need to use to increase the current range n times, i.e. that $I = nI_m$. On the basis of Kirchhoff's Second Law, we

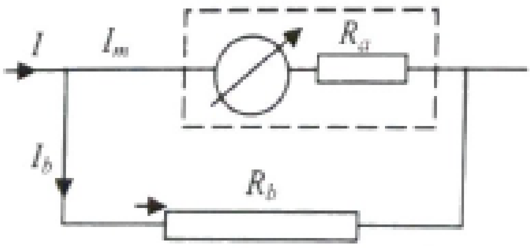


Figure 11.2. Shunting the ammeter

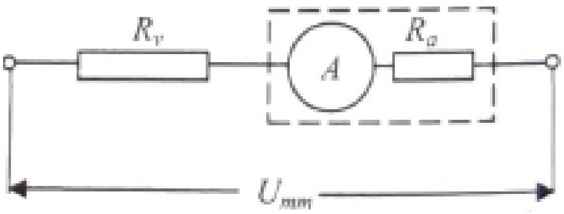


Figure 11.3. Internal resistance of the voltmeter

find the proportion

$$\frac{R_a}{R_b} = \frac{I_b}{I_m}, \tag{11.7}$$

where R_a is the internal resistance of the ammeter, R_b is the resistance of the shunt.

According to Kirchhoff's law I $I_b = nI_m - I_m = I_m(n-1)$. Taking into account the higher relation in formula (11.8), we can calculate the shunt resistance.

$$R_b = \frac{R_a}{n-1}. \tag{11.8}$$

Thus, in order to increase the range n-fold, a shunt with a resistance $(n-1)$ times smaller than the internal resistance of the ammeter should be used. In *multi-range meters*, the shunts are built inside and they are switched on by means of multi-position switches.

Voltmeter

We can adapt any current measuring device to the voltage measurement by attaching an appropriate high resistance in series to it. Suppose that the maximum current of the ammeter is still I_m and we want to get a volt of measure in the range U_m. For this purpose, we need to add a resistance R_w in series to the ammeter (Fig. 11.3) that at the voltage U_m the current flowing is equal to I_m

$$I_M = \frac{U_m}{R_w + R_a}. \tag{11.9}$$

where we calculate the resistance of the voltmeter

$$R_w = \frac{U_m}{I_m} - R_a. \tag{11.10}$$

The current flowing in the voltmeter branch, which we turn on in parallel with the source or other circuit element, should be as low as possible. We achieve this by using a sensitive microammeter combined with high resistance.

Electronic meters

A relatively new group of devices are electronic meters, in which the basic operation is voltage measurement by comparing it with the reference voltage. The internal resistance of these devices is very high (reaching 10^{11} ohms), which makes them extremely widely used. In many types of instruments, the result is given in a digital form, which makes reading easier and increases its accuracy.

12. The use of a computer in measurements

Introduction

The properties of computers, including personal computers, allowed them to be widely used to measure various physical quantities. Particularly useful are properties such as the speed of measurement, the ease of repeated repetition, storage and processing of the measured data.

Computer is a digital device, ie it performs operations on quantized quantities - assuming only specific values that may differ by a multiple of a certain constant. The opposite of a quantum quantity is a continuous quantity, also called an analog quantity, which can take any value, and the difference between the different values can be as small as you like.

EXAMPLE

An integer is a digital quantity. The range of integers from 0 to 10 contains only 11 values, and the smallest difference between them is 1. The real number is continuous. Within the same range [0.10] there are an infinite number of values, e.g. 3.1415927 and 3.1415928, which can be anywhere close to each other.

In a computer, as well as in other digital devices, the digital quantity is represented by a series of voltage pulses, which can only take two values: 0 V and 5 V. From these pulses an integer is made in such a way that 0 V is assigned the value 0, and for the 5V pulse - the value 2^n, where the n exponent represents the position of a given pulse in the sequence. Creating a digital value using eight pulses is shown in Fig. 12.1.

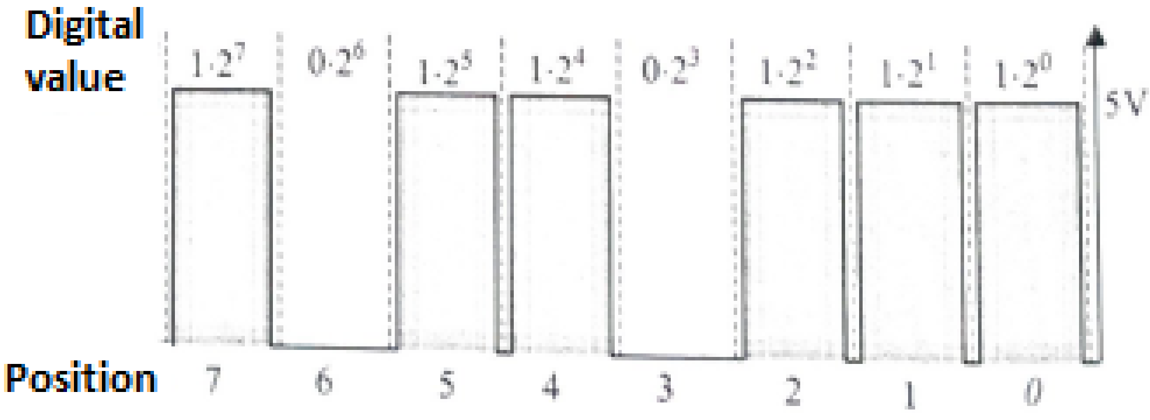


Figure 12.1. Creating a digital value from a series of square pulses; each pulse is assigned the value $i\ cdot 2^n$ ($i = 0$ for 0 V, $i = 1$ for 5 V, n - pulse position. The total value represented in the figure is 183 (in decimal) or 10110111 (binary)

In order to use a computer for measurement purposes, we need to provide a digital signal to it, while the measured physical quantities are continuous quantities. Adapting various physical quantities to digital form requires two transformations:

- conversion of a given quantity into electric voltage,
- converting voltage (analog) to digital value using the so-called analog-to-digital converter.

The analog-to-digital converter, apart from its essential function, also has a time counting function.

Computer measuring set

The *Science Workshop* measuring system is used in the Physical Laboratory of the Poznań University of Technology. The system consists of three main components which process the information sequentially as shown in the diagram below.

Two types of sensors are used - analog and digital. *The analog sensor* produces a voltage proportional to the physical quantity. For example, a temperature sensor produces a voltage proportional to temperature - at 0°C it generates 0 V; when the temperature is 10°C, the voltage is 0.1 V, and at 100°C it is -1 V. The analog voltage from the sensor is converted by the interface into a digital signal which is in turn passed to the computer.

The *digital sensor* produces a different type of signal. It produces only two voltage values - 5 V and 0 V. An example of such a sensor is the so-called the photo frame is presented schematically in Fig. 12.2. One of the gate's arms has a light source, and the other - a detector that generates a voltage of 5 V under the influence of lighting. Covering the beam causes the signal to drop to 0 V.

A photo gate can be used to measure the transit time of an object or the time interval between the passes of two objects. The timing accuracy is 0.0001 s so 0.1 ms.

The interface is a device that is connected to the sensor on one side and to the computer on the other. The front face of the interface is visible in figure 12.3 as a dark element at the top of the drawing. On the front panel there are sockets for connecting digital sensors, marked 1 and 2, and sockets for analog sensors A, B, C. Before starting measurements, we must attach a sensor appropriate to the measurement to the interface. In addition to the physical connection, it is necessary to "inform" about this program by indicating which sensor has been connected and to which input (channel). This is done by pressing the mouse button when the cursor points to the sensor plug, dragging the cursor to the appropriate interface socket.

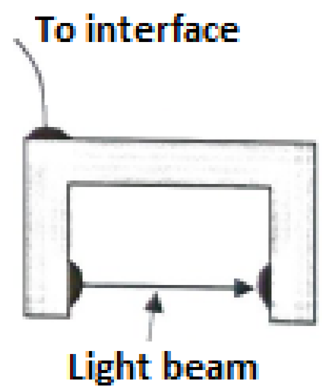


Figure 12.2. Photogate

Measure handler

Performing a measurement with the use of a computer requires appropriate software, which enables e.g. starting and ending the measurement, determining the necessary parameters, making calculations or presenting the results. The user communicates with the computer by means of buttons, selection lists and data entry windows. The communication elements are adapted to the actual situation of the measurement process.

Figure 12.3 shows *the settings window* after starting the program. In addition to the sensor plugs and sockets already mentioned, there is a set of possible ways of presenting the measurement results. It is also possible to call up the calculator and notebook on the screen. In the upper left corner there are buttons for controlling the measurement process - *"Zapis"* - *Record*, *"Podgląd"* - *Preview* and *"Stop"* - *Stop*:

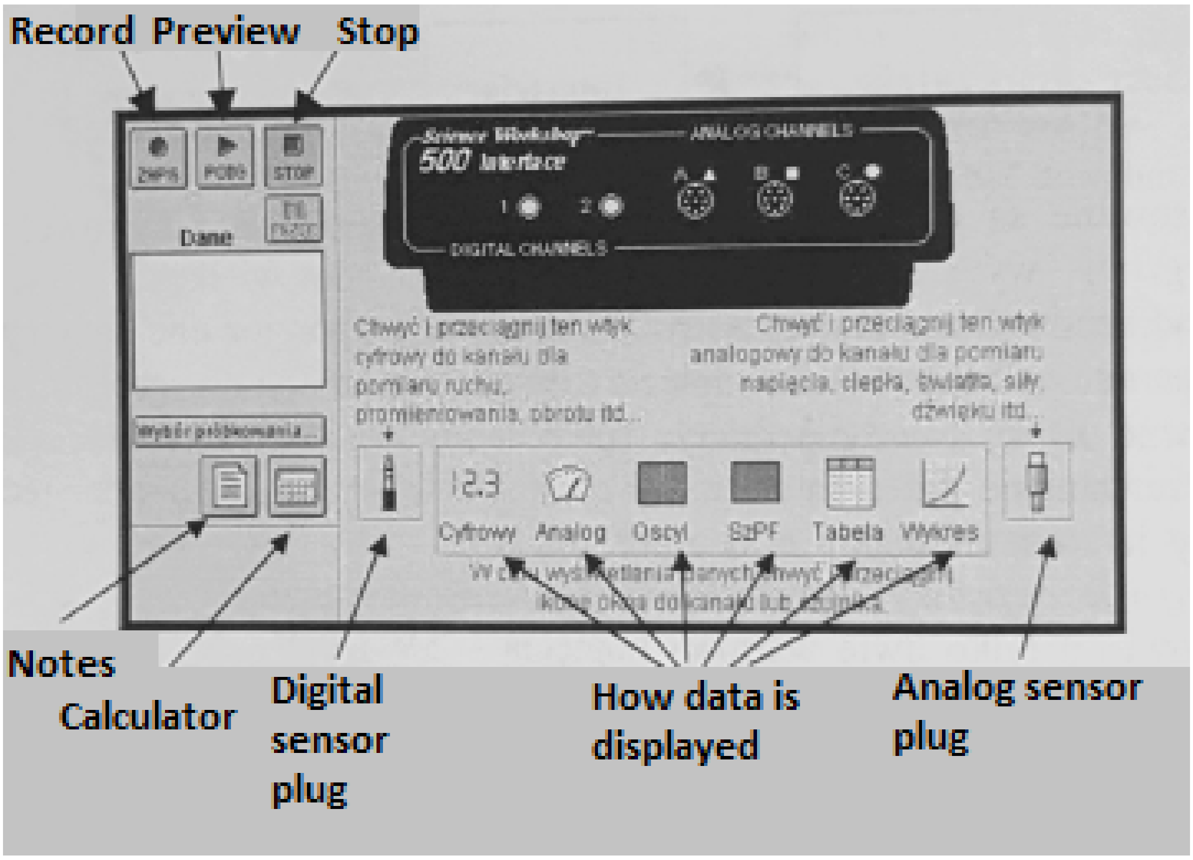


Figure 12.3. Settings window

- Clicking on *"Zapis"* - *Record* starts recording the measurements, which are simultaneously displayed on the screen and saved in the memory.
- Clicking on *"Podgląd"* - *Preview* performs the same actions, except for saving to memory - starting another measurement causes the loss of previous data.
- The *"Stop"* - *Stop* button is used to stop recording measurements.

After the desired measurements have been taken, they can be presented in various ways. When the result of the measurement is a single value or a series of slowly changing values, then it makes sense to show the results in digital or analog form - just as if we were using a digital or dial gauge.

When the result of the measurement is a series of values that vary over time, it is more convenient to present them in the form of a table or graph. You can create a table or chart window in two ways:

- from the settings window (fig. 12.3) - grabbing the table or graph icon with the mouse and dragging it to the selected slot,
- from the main menu - by selecting *"Wyświetl"* - *View* and then *"Nowa tabela"* - *New table* or *"Nowy wykres"* - *New chart* from the drop-down list.

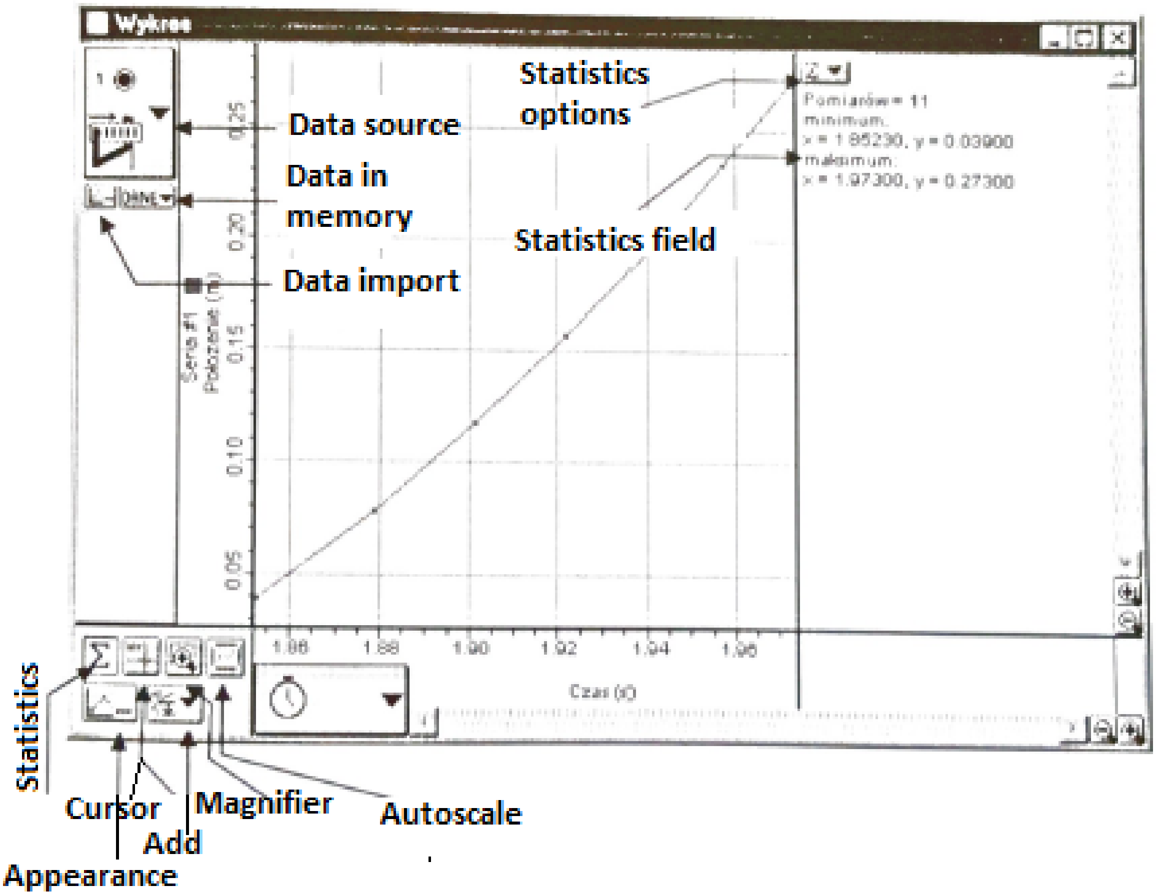


Figure 12.4. Settings window

An example of a chart window is shown in fig. 12.4. The main part of the window contains the actual graph and the statistics field. In addition, there are content control buttons:

- *"Źródło danych"* - *Data source* - allows you to define or change the channel from which the data is to be downloaded and the type of data from the selected channel. For example, you can select digital channel 1 (pl. *"kanał cyfrowy 1"*) and digital channel 2 (pl. *"kanał cyfrowy 2"*) after selecting a channel *"Położenie"* - *Position*, *"Prędkość"* - *Speed* and *"Przyspieszenie"* - *Acceleration*. More than one type of data can be selected; then the window will contain more graphs or columns table,
- *"Dane w pamięci"* - *Data in memory* - allows you to download data previously measured and stored in memory.
- *"Import danych"* - *Data import* - concerns downloading data saved in a different format (by another program).
- *"Statystyka"* - *Statistics* - opens and closes the statistics field in the right part of the window, one by one.

- *"Kursor"* - *Cursor* - makes a cross that moves with the movement the mouse and entering the coordinates at the cursor position.
- *"Lupa"* - *Magnifier* - magnifies a fragment of the chart.
- *"Autoskalowanie"* - *Autoscale* - adjusts the chart coordinate range to the range measured quantities.
- *"Wygląd"* - *Appearance* - allows you to set graph parameters, such as mesh, connecting points.
- *"Dodaj"* - *Add* - allows you to add the next chart with the current one.

The content of the *statistics field* is defined after clicking the *"Opcje statystyki"* - *Statistics Options* button. Figure 12.4 shows the number of measurements, minimum and maximum values in this field. Further possible statistical data are mean values and standard deviations. A separate group includes: function fit, derivative, integral and histogram.

Function fitting consists in finding the parameters of the function $y(x)$ best suited to the measurement data. For example, matching a linear function is finding the parameters a_1 and a_2 of the function $y = a1 + a2x$. After selecting *"Dopasowanie krzywej"* - *Curve fit* and then *"f. liniowa"* - *linear function*, the statistics field shows the form of the function from the previous sentence and the values of the parameters a_1 and a_2. The markings remain the same, regardless of the variable names that are currently being measured. For example, when examining uniformly accelerated motion, we have the function $v = vo + at$. In this case, the statistic results have the following meanings: $x = t$, $y = 0$, $a_1 = v_0$, $a_2 = a$.

From the data in the table or in the chart, we can select only a certain range and enclose it inside the rectangle circled with the mouse that appears as a dimmed area. All calculations in the statistics field refer to the selected range. Not selecting is the same as selecting all data.

4. Mechanics

13. Determination of the speed of sound in the air by the phase shift method

Introduction

The sound propagates in a mechanical wave and occurs only in the elastic medium. If a certain element of the medium in which the particles are connected, we stimulate vibrations, the energy of vibrations of this element will be transmitted to neighbouring elements and will cause them to vibrate.

The vibrations propagating in the center are called a *wave*. In the wave motion, it can be noticed that the medium does not follow the propagating wave but vibrate around its equilibrium position. If the direction of vibration and wave is parallel, then the wave is called *longitudinal*, while when the vibrations of particles are perpendicular to the direction of wave propagation, the wave is called *transverse*.

The nature of the wave propagating in the medium depends on its elastic properties. If elastic forces arise as a result of shifting the layers of the medium, striving to restore the layer shifted to the equilibrium position, then transverse and longitudinal waves may be formed in the medium. Most often this happens in solids. However, if there is no elastic force between the displaced layers, only longitudinal waves are propagated. This happens in liquids and gases.

Harmonic motion is the most common *oscillatory motion* in which the y-variation changes at time t according to the equation:

$$y = A\sin(\omega t + \phi_0), \tag{13.1}$$

where: A- amplitude, ω - circular frequency, ϕ_0 - initial phase.

The expression $(\omega t + \phi_0)$ is the *phase* of oscillation motion. In practice we find the phase of each motion as an angle, for which plot of not shifted sine function has the same inclination state. For example, decreasing deflection by values $A/2$ has a phase equal to $5/6\pi$ or $150°$.

The *initial phase* is defined as the state of motion at time $t = 0$ and it is selected arbitrarily. Taking for example $\phi = 0$, we assume, that in time $t = 0$ oscillating point passing through the equilibrium position for the side of positive inclination. The phase

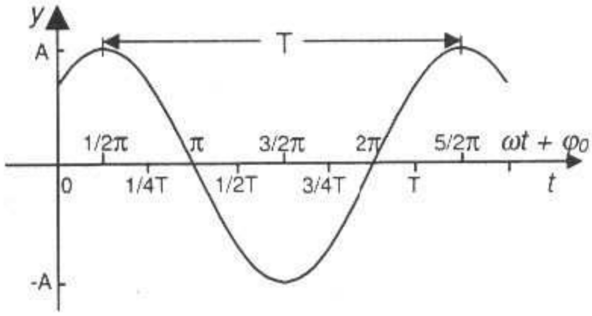


Figure 13.1. Deflection in harmonic motion as a function of time (t) or phase ($\omega t + \phi_0$)

is expressed in angle units (degrees or radians). An example of a harmonic motion diagram with some phases marked is shown in (Figure 13.1).

The velocity of the wave propagation is called the speed of the constant phase deflection. The deflection y of any moment t, at a distance x from the source of vibration is described by the wave function:

$$y = A\sin(\omega t - kx - \phi_0), \tag{13.2}$$

where: $k = 2\pi/\lambda$ - wave number, λ- wavelength, ϕ_0- phase at point $x = 0$ and at time $t = 0$. The wave phase is the expression $(\omega t - kx - \phi_0)$. Wave equation is doubly periodic, relative to time and space. The relationship between the period T and the length of the wave can be found in the consideration of the motion of a constant phase deflection. The phase stability is described by the equation:

$$\omega t - kx - \phi_0 = const. \tag{13.3}$$

We calculate from the equation (13.3) the value of x:

$$x = \frac{\omega t - \phi_0}{k}, \tag{13.4}$$

and then differentiating with respect to time t, we get the formula for the wave speed:

$$v = \frac{dx}{dt} = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = \lambda \cdot f, \tag{13.5}$$

where T - wave period, f - frequency of acoustic wave.

Thus, the wavelength is the path travelled by the wave during one period.

Acoustic waves can propagate in solids, liquids and gases. Acoustic waves with frequencies ranging from 20 Hz to 20,000 Hz are called audible waves, because they evoke auditory sensations in the human brain. The sources of audible waves are vibrating strings (e.g. violin, guitar, human vocal cords), vibrating columns of air (e.g. pipes, organs, clarinet) and vibrating plates and membranes (e.g. drum, loudspeaker).

these vibrating objects alternately condense and dilute the surrounding air, causing the particles to move back and forth. air carries this disturbance from its source into space.

The perception of the perceived sound is determined by its intensity, pitch and color. The aforementioned features of sound depend on the appropriate wave parameters - amplitude, frequency and harmonic vibration content.

Wave speed in the air

The speed of propagation of longitudinal waves in a continuous medium can be given by the formula:

$$v = \sqrt{\frac{E}{\rho}}, \tag{13.6}$$

where E - Young's modulus of the medium, ρ - its density. By converting the Hooke's law, you can write:

$$E = \frac{-dp}{\frac{dV}{V}}, \tag{13.7}$$

dp and dV differential changes of pressure and volume of gas of volume V.

Sound vibrations propagate so fast that compression and thinning of the gas can be considered adiabatic, so the change in the gas state follows the Poisson formula:

$$pV^{\kappa} = const, \tag{13.8}$$

κ - ratio of specific heat at constant pressure to specific heat at constant volume (for 2-atom gases $\kappa = 1.4$).

Differentiating Poisson's formula we obtain:

$$V^{\kappa}dp + \kappa V^{\kappa-1}pdV = 0, \tag{13.9}$$

and from here

$$\frac{dp}{\frac{dV}{V}} = -\kappa p. \tag{13.10}$$

After substituting the obtained value into equation 13.7 and then taking into account the Young modulus form obtained in this way in equation 13.6, we express the longitudinal wave velocity by the formula:

$$v = \sqrt{\frac{\kappa \cdot p}{\rho}}. \tag{13.11}$$

To eliminate the density ρ, we take the definition of this quantity and multiply the numerator and denominator by p - gas pressure. Note that in the denominator we have the product of pV, which we can replace with the product of nRT based on the ideal gas equation:

$$\rho = \frac{m}{V}\frac{p}{p} = \frac{mp}{nRT}, \tag{13.12}$$

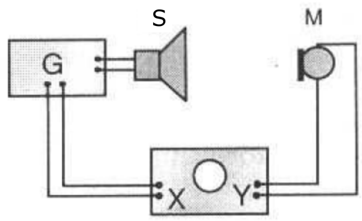


Figure 13.2. Block diagram of the electrical system; G - sinusoidal voltage generator, M - microphone, S - speaker, X, Y, - oscilloscope inputs

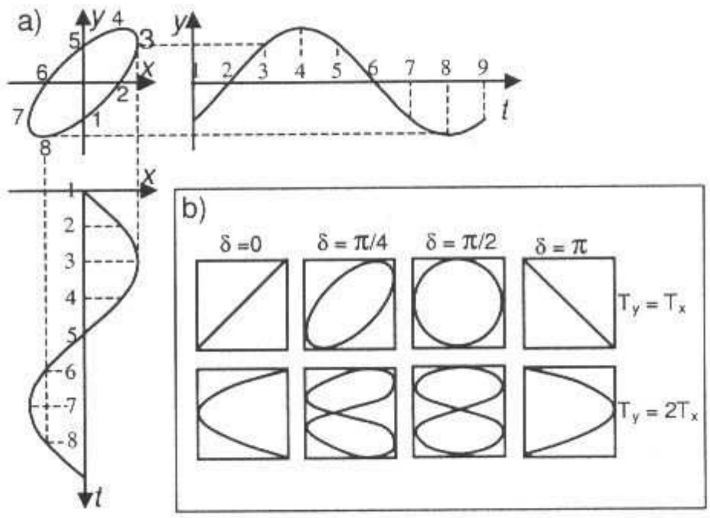


Figure 13.3. Construction of Lissajous figure for phase difference 45° (a) and examples of Lissajous figures for other phase difference and periods (b)

where: m -mass of gas, n- amount of gas in moles , R- universal gas constant, T - temperature.

The number of moles n can be expressed as the ratio of the total mass of gas to the mass of 1 mole μ: $n = m/\mu$. Taking this into account in the above equations, we get a formula for *the speed of sound*:

$$v = \sqrt{\frac{\kappa RT}{\mu}}. \tag{13.13}$$

The principle of measurement

The aim of the exercise is to determine the speed of sound v in the air from the formula:

$$v = \lambda f, \tag{13.14}$$

where λ is the wavelength, and f is frequency of sound wave.

The frequency f is measured directly by the frequency meter, while to determine the wavelength we will use the method explained below.

At one end of the horizontal bench we place the loudspeaker connected to the electric vibration generator acting as the source of the waves (Figure 13.2). To pick them up we use a microphone moved on a movable cart along the bench.

We apply the speaker voltage to the X oscilloscope plates, and the microphone voltage to the Y plates. Both voltages change over time and reflect the movement of vibrating air particles. on the oscilloscope screen we will receive an image in the form of *figur Lissajous* (Figure 13.3).

The Lissajous figure arises as a result of overlapping two harmonic movements whose vibration directions are perpendicular to each other. Its shape depends on the difference between the phases of the component vibrations and the ratio of their frequency. In the exercise the vibration frequency is equal, so the shape of the figures is determined only by the phase difference between the speaker and the microphone.

The shape of the Lissajous figure is a periodic function of the phase difference, with a period of 2π, or it will be the same for all microphone settings that differ by the total multiple of the wavelength.

The sound wave length is measured using the above property. We determine the distances between neighboring positions of the microphone, for which we obtain the same shape of the Lissajous figure. The most accurate results are when the figure is a straight line with the same slope (0, 2π, 4π, etc.).

When we know the wavelength and read the frequency from the meter, we calculate the speed of the sound using the formula 13.14. In order to obtain a more accurate result, we perform measurements for various frequency values. Finally, we compare the result with the speed of sound obtained from the formula 13.13. To do this, measure the room temperature during measurements.

The exercise aims at determining the speed of sound in the air. The experimental setup is a wave generator connected to a speaker as well as a microphone (at a regulated distance from the speaker) used to receive the signal.

Experimental Procedure:

1. Connect the electrical system as per (Figure 13.1)
2. Start the acoustic generator and set the selected frequency (in the range from 3 kHz to 8 kHz).
3. Start the oscilloscope.
4. On the oscilloscope, set the image about the size of about half the size of the screen.
5. By changing the microphone distances from the speaker, in the entire range, find the positions in which the image is a straight line with the same sign of the slope factor.
6. Estimate if the distances between consecutive positions are the same (approximately). If not, the measurements should be rejected.
7. Calculate the average difference in microphone positions and wavelength.
8. For the calculated wavelength, calculate the speed of sound from the equation 13.14.
9. Calculate the speed of sound for several other frequencies.
10. Calculate the average speed of sound and the standard deviation of the mean.
11. Calculate the speed of sound based on the equation 13.13 and compare with the experimental result.
    Note: The ratio of specific heat at constant pressure to specific heat at constant volume for air is: $\kappa = 1.4$.
12. Write down the final conclusions

Keywords:

- Mechanisms of wave propagation in an elastic medium, longitudinal and transverse waves
- Harmonic movement: time dependence of swing, phase, initial phase
- Wave motion: the dependence of deflection on time and space, period, wave length, wave speed
- Acoustic waves, sound features
- Hooke's law, adiabatic transformation, Poisson equation, deriving the formula for the speed of sound in perfect gas
- Wavelength measurement of phase shift
- Construction of Lissajous figures

14. Determination of gravitational acceleration using a reversible and mathematical pendulum

Intorduction

Physical and mathematical pendulums oscillate under the influence of gravity. In the range of small amplitudes, this motion is a harmonic motion, its period depends on the properties of a given pendulum and the gravitational acceleration.

A physical pendulum is any rigid body that can swing around its horizontal axis. After swinging out of equilibrium, the moment of gravity $mgL\sin\varphi$ acts on the body (Fig. 14.1). By applying the second law of dynamics to this situation, we obtain the equation:

$$-mgL\sin\varphi = I\frac{d^2\varphi}{dt^2}, \tag{14.1}$$

where: I - moment of inertia of the body with respect to the suspension point A, φ - angle of deflection from the equilibrium position, L - distance from the suspension point A to the center of gravity C. The minus sign in equation (14.1) indicates that the effect of the moment of force always results in a decrease in the deflection of the body.

The basic property of harmonic motion is that the acceleration is proportional to deflection (linear or angular). Taking the above into account, we note that the motion of the physical pendulum in general is not a harmonic motion - equation (14.1) shows that the angular acceleration is proportional to the sine of the deflection angle, and not to the angle itself. Only the criterion of harmonic motion will be met in the range of small swings for which $\sin\varphi = \varphi$. If we limit ourselves to small deflec-

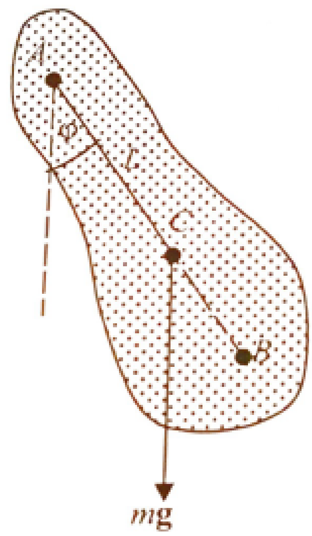


Figure 14.1. Physical pendulum; A, B - suspension points, C - center of mass

tions (a few degrees), the equation (14.1) can be written as:

$$\frac{d^2\varphi}{dt^2} = -\frac{mgL}{I}\varphi. \tag{14.2}$$

The general equation of harmonic motion takes the form:

$$\frac{d^2\varphi}{dt^2} = -\omega^2\varphi, \tag{14.3}$$

where ω is the angular velocity. As a result of comparing the last two equations, we obtain the expression that determines the period of the physical pendulum:

$$T_f = 2\pi\sqrt{\frac{I}{D}}, \tag{14.4}$$

where $D = mgL$ is the steering moment. A mathematical pendulum differs significantly from the physical one in mass distribution - it is a material point suspended on a weightless thread. If we denote the thread length by l, the period of the mathematical pendulum's vibrations is given by the formula:

$$T_m = 2\pi\sqrt{\frac{l}{g}}, \tag{14.5}$$

Imagine that we have a specific physical pendulum, as well as a mathematical pendulum with an adjustable length. It is obvious that we can choose the length of the latter so that the periods of vibration of both pendulums are equal. Thus, we have determined the reduced length of the physical pendulum. It is equal to the length of a mathematical pendulum having the same period.

We calculate the reduced length l_r by comparing equations (14.4) and (14.5) with each other.

$$l_r = \frac{I}{mL}. \tag{14.6}$$

If we know the reduction of a physical pendulum, then its oscillation period can be found using the equation for the mathematical pendulum; It is not necessary to know either the moment of inertia or the steering moment.

$$T_f = 2\pi\sqrt{\frac{l_r}{g}}, \tag{14.7}$$

To determine the reduced length we use the following property of the physical pendulum: if the period of the suspended pendulum at A is equal to the period of the suspended pendulum at B (Fig. 14.1), then the distance between the suspension points is the reduced length.

To demonstrate this property, we will first assume that the distance between the suspension points ($AB = l$) is arbitrary, and then we will find the conditions for which T_A and T_B are possible.

$$T_A = 2\pi\sqrt{\frac{I_A}{mgL}}, \quad T_B = 2\pi\sqrt{\frac{I_B}{mg(l-L)}}. \tag{14.8}$$

Moments of inertia with respect to the axes passing through the points A and B are weighed by the moment I_c with respect to the parallel axis passing through the center of gravity. On the basis of *Steiner theorem*, the corresponding moments are determined by the expressions:

$$I_A = I_C + mL^2,$$
$$I_B = I_C + m(l - L)^2. \tag{14.9}$$

After taking into account the above relationships, we bring the condition of equal periods $T_A = T_B$ to the form:

$$\frac{I_C + mL^2}{L} = \frac{I_C + m(l - L)^2}{l - L}. \tag{14.10}$$

This is a quadratic equation for l. Solving them in an elementary way, you will get two solutions:

$$l_1 = 2L, \tag{14.11}$$

$$l_2 = \frac{I_C + mL^2}{mL} = \frac{I_A}{mL}. \tag{14.12}$$

The value of l_1 corresponds to the case when both suspension points are symmetrical to the center of gravity, while l_2 is just the reduced length, which we can find by comparing the last equation with equation (eqn:e102.6).

The above reasoning proves that indeed, if the periods of a pendulum suspended at different points are equal, the distance between these points is the reduced length of the physical pendulum.

A special form of a physical pendulum, facilitating the determination of the reduced length, is a reversible or reversible pendulum, the structure of which is shown in Fig. 14.2.

There are two axes A and B on the metal rod, serving as suspension points and at the same time the oscillation axes and metal weights in the form of S_1 and S_2 lenses. Axes and lenses can be moved along the rod. The position of the center of mass and the moment of inertia depend on the position of the lenses. The pendulum is not reversible for any position of the axis and lenses - the oscillation periods for both suspensions are different. By manipulating the position of the lens, we can make the two oscillation periods equal, and only then is the distance between the axes a reduced length.

Figure 14.2. The reversible pendulum; A, B - suspension points, S_1, S_2 - movable lenses

Measurements:

A. The mathematical pendulum

1. Mount the photoelectric sensor so that its mathematical pendulum ball is sealed.
2. Take measurements for several lengths (l) of the pendulum, measure the duration of ten swings (t_m) at least three times.
3. Note the accuracy of the time measurements (Δt) and pendulum length (Δl).

B. The reversible pendulum

1. The reversible pendulum has two axes (A, B) and two lens-shaped weights (S_1, S_2). Measure the time of 10 pendulum cycles hanging on the A axis and calculate the period T_A.
2. Systematically change the position of the S_1 weight by 5-10 cm along the $A - B$ distance. At each setup measure the the T_A period.
3. Invert the pendulum by hanging it on the B axis.
4. Repeat the measurement of the T for B axis (T_B periods) by changing the position of the S_1 lens every 5–10 cm in the entire range of positions between the A and B axes.

Report:

A. The mathematical pendulum

1. Calculate the periods of vibration of the mathematical pendulum ($T_m(l)$ - based on time measurements).
2. Knowing the lengths of the mathematical pendulum and the period of its fluctuations, calculate (for each length) the value of gravitational acceleration ($g_m(l)$, $T_m = 2\pi\sqrt{\frac{l}{g}}$).
3. Based on the results obtained, calculate the average acceleration of gravity determined using a mathematical pendulum (g_m).
4. Calculate the uncertainty of this acceleration (Δg_m) and present the final results of the experiment (properly rounded off).

B. The reversible pendulum

1. Plot the periods T_A and T_B as a function of the S_1 weight position (both curves on the same graph).
2. The crossing point of the curves represents the T period identical for both setup. The point of intersection of T_A and T_B curves determines the period T, the same for both suspensions. **Note:** The second coordinate of the intersection is not a reduced length!
3. Estimate the error of reading the intersection point on the graph. This is also an error in determining the period.

4. Based on the determined reduced length[1] l_r, calculate the gravitational acceleration g from (eq. 14.7): $T = 2\pi\sqrt{\frac{l_r}{g}}$
5. Calculate the error g - the easiest way using the logarithmic differential method.
6. Round off and compile the results and their errors.
7. Write down the final conclusions

Keywords

- Simple and damped harmonic motion: acting forces, differential equation, time dependence of the deviation
- Second law of dynamics
- Physical pendulum: center of gravity, moment of force, moment of inertia, the period of fluctuations, length reduced
- Steiner theorem (see also chapter 16 and 20)
- Reversible pendulum; measurement activities

15. Determination of the linear expansion coefficient of solids

Introduction

A change in body temperature is usually accompanied by a change in its linear dimensions, and thus also a change in volume. Elementary temperature rise dT of the body, whose length is l, causes an increase in length l by dl given by the formula:

$$dl = \alpha l dT. \tag{15.1}$$

The α factor is called the linear expansion coefficient. Its numerical value is the relative length increment dl/l caused by a temperature change of 1°C and depends on the type of body and also on the temperature.

Due to the dependence of the α coefficient on temperature, body length is generally a non-linear function of temperature. Within the scope of slight changes in temperature, it can be approximately assumed that the coefficient α is constant and the length increases in direct proportion to the temperature. In this case, the equivalent of formula (15.1) is the following formula:

$$l - l_0 = \alpha_{sr} l_0 \Delta T, \tag{15.2}$$

which allows a simple length calculation at any temperature.

The causes of the phenomenon of thermal expansion are to be found in the microscopic structure of bodies. Solids are made of atoms (ions) distributed regularly in space and forming *a crystal lattice*. The atoms are bound together by electrical forces, which prevents them from permanently changing their position. The thermal energy

[1] A reduced length of a physical pendulum l_r is the length at which a reversible pendulum placed on A or B axis has the same period. For used in this exercise reversible pendulum the distance between A and B axis is the reduced length $l_r = 0.87 \pm 0.01$ [m].

supplied to the crystal causes the atoms to vibrate around the equilibrium positions. The amplitude of these vibrations increases with temperature. The thermal oscillation frequency of the atoms is up to 10^{13} Hz. In this situation, the concept of interatomic distance only makes sense as the distance between the centers of vibration of adjacent atoms.

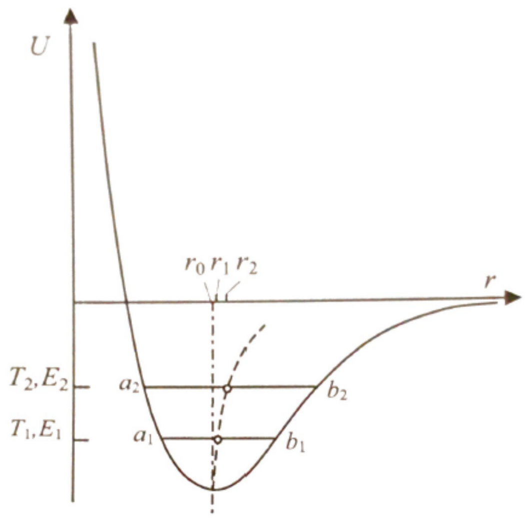


Figure 15.1. The potential energy of two atoms as a function of their mutual distance

The potential energy of two interacting atoms as a function of the distance between atoms is given by the curve presented in Fig. 15.1.

If the kinetic energy of the atoms were zero, they would be at a distance of r_0 from each other, and at this distance, the potential energy has a minimum. In fact, atoms vibrate around equilibrium positions, i.e. they have a specific kinetic energy that increases with increasing temperature. At T_1, the distance between atoms varies from a_1 to b_1. Due to the asymmetry of the potential curve, the mean position of the vibrating particle will not coincide with the value of r_0, but will shift to the right and will reach the value of r_1. After increasing the temperature to T_2, the system will move to a higher energy level E_2 - the distance will change from a_2 to b_2, and the average position will reach value r_2.

The above description shows that with increasing temperature, not only the vibration amplitude of atoms increases, but also the average distance between them, which macroscopically manifests as thermal expansion.

Analogously to the linear expansion coefficient, we define *the volumetric expansion coefficient*:

$$\gamma = \frac{dV}{V_0 dT}. \tag{15.3}$$

The body volume when heated by ΔT is given by the formula:

$$V = V_0(1 + \gamma_{sr}\Delta T). \tag{15.4}$$

In order to find the relationship between α and γ, consider a cube whose edges increase in length according to equation (15.2). The volume of the cube depending on the temperature can be expressed as:

$$l^3 = l_0^3(1 + \alpha_{sr}\Delta T)^3. \tag{15.5}$$

The binomial cube expansion contains the product $\alpha\Delta T$ in the first, second and third powers. Since this product is small relative to one, the second and third powers of this product are very small and we can omit them in the expansion of the binomial cube. In view of the above, the equation (15.5) can be written as:

$$l^3 \approx l_0^3(1 + 3\alpha_{sr}\Delta T). \tag{15.6}$$

Comparing the last equation with the equation (15.4), we come to the conclusion that

$$\gamma \approx 3\alpha. \tag{15.7}$$

The value of the linear expansion coefficient in polycrystalline and amorphous bodies does not depend on the direction, while in single crystals (anisotropic bodies) the dependence on the direction is clear - instead of one, there are three main linear expansion coefficients, determined for the three crystallographic axes of the crystal.

Principle of measurement

Place the tested object in the form of a rod in a water jacket (Fig. 15.2) after it is connected to the thermostat. One end of the rod is fixed in the handle, while the other end moves as it is heated. The rod elongation is measured with a micrometric sensor, and the rod temperature is measured with an electronic thermometer or a thermocouple and a millivoltmeter. In order to calculate the coefficient of expansion from the measured data, we will write the equation (15.2) as:

$$\Delta l = \alpha_{sr} l_0 T - \alpha_{sr} l_0 T_0, \tag{15.8}$$

where T_0 is the initial temperature at which the bar length is l_0.

Equation (15.8) means that the elongation is a linear function of temperature and that the slope of the line is

$$a = \alpha_{sr} l_0. \tag{15.9}$$

We compute the value of a by applying linear regression to data pairs (Δl, T). If we also measure l_0, then equation (15.9) can be used to finally calculate the expansion coefficient.

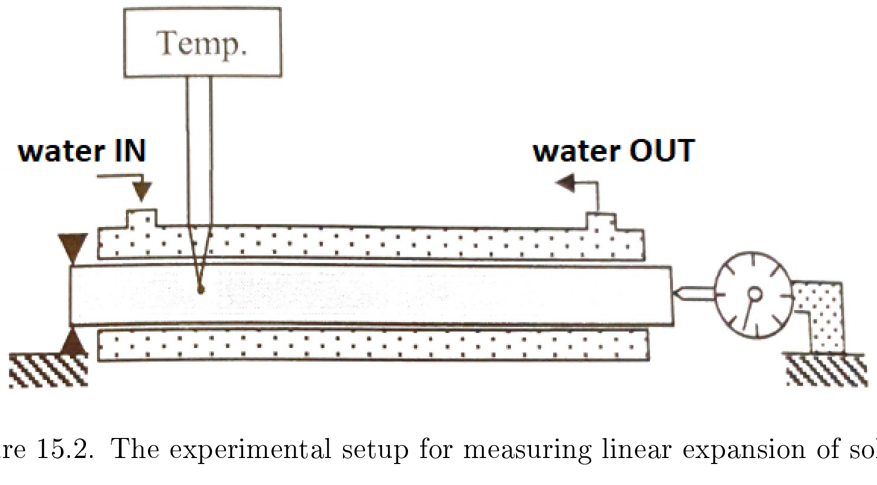


Figure 15.2. The experimental setup for measuring linear expansion of solids

Measurements:

1. Measure the initial length of given solid rods by using the built-in caliper (see 5).
2. Read off the initial temperature.
3. Heat the rods by using the ultrathermostat and according to the instructions given in class (for details see section 7).
4. At each temperature (every 3-5 deg. Celsius) – record the exact temperature and the length increase (dl) of all the rods by using the micrometer ((see 5).
5. When the final temperature is reached (approximately 70 deg. Celsius), continue the measurements while cooling the system down.

Report:

1. Plot the elongation as a function of temperature and determine the slope coefficient.
2. Calculate the linear expansion coefficient α from the relation: $a_{reg} = \alpha \cdot l_0$, where a_{reg} is the slope coefficient and l_0 is the initial length.
3. Calculate the error, the easiest way is using the logarithmic differential method.
4. Present the final form of the results and errors after rounding.
5. Write down the final conclusions

Keywords:

- Length change with elementary temperature change, coefficient of linear and volumetric expansion
- Length and volume at any temperature. The influence of temperature on the vibration amplitude and the distance between atoms, the potential energy of interaction of two atoms
- Expansion of anisotropic bodies
- Temperature measurement
- Vernier, ultrathermostat (for details see section 5 and 7).

16. Investigation of the moment of inertia

Introduction

In the description of the dynamics of the translational movement, the notion of inertia associated with mass appears in moving bodies. In the case of rotational movement, the knowledge of body mass is insufficient, and its spatial distribution relative to the axis of rotation is also important. Physical parameter containing information on body mass and its spatial distribution relative to the axis of rotation is moment of inertia I. This quantity appears in the principles of the dynamics of the rotary motion, in principle the conservation of momentum, etc. For a single point mass with mass m rotating around an axis distant from it by a distance of r (fig. 16.1a) we can get the following relationship for the moment of inertia:

$$I = mr^2. \tag{16.1}$$

In the case of a system N point masses rigidly connected to one another in relation to the axis of rotation, called the axis of inertia (Fig. 16.1b), the moment of inertia of the system is equal to the sum of moments of inertia of each point mass:

$$I = m_1 r_1^2 + m_2 r_2^2 + ... + m_N r_N^2 = \sum_{i-1}^{N} m_i r_i^2, \tag{16.2}$$

where m_i is a mass and i - this point mass, and r_i its distance from the axis of inertia. If we are dealing with a rigid solid with a mass M, we hypothetically divide it into a set of infinitely small elements (sections) with masses dm. The moment of inertia of a solid is equal to the sum of moments of inertia of individual elements. Assuming that mass dm element of the solid tends to zero, the sum can be written in an integral form:

$$I = \int_0^M r^2 dm, \tag{16.3}$$

where r is the distance of the element with mass dm from the rotation axis (fig. 16.1c). Calculation of the moment of inertia on the basis of formula (16.3) are relatively simple

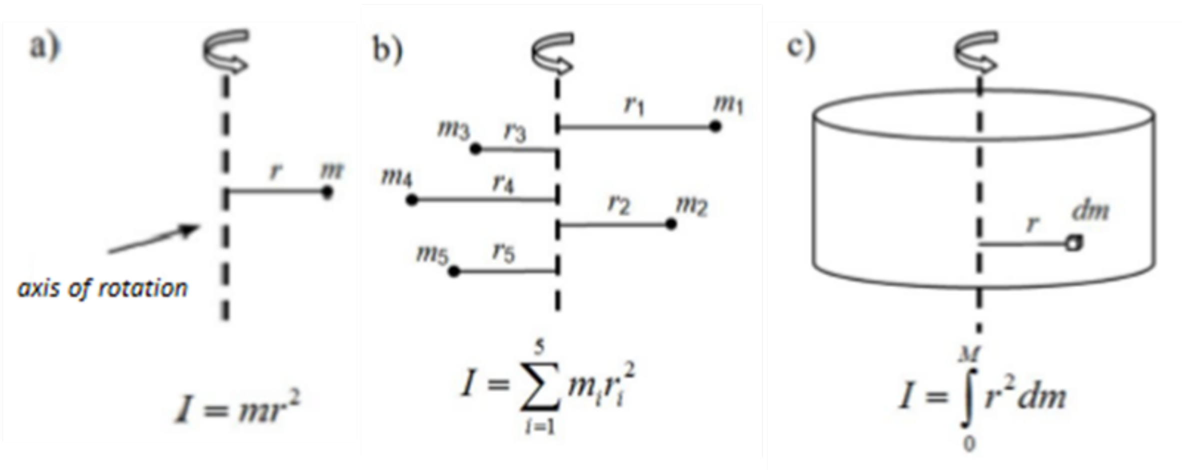


Figure 16.1. Visualization determining moment of inertia for a) a point mass b) the set of point mass rigidly connected to one another, c) the rigid body.

only for solids having symmetry axis parallel to the axis of inertia (rod, cylinder, ball, etc.). In the case of solids with a complex or irregular shape, analytical methods are very complicated. In practice, the moment of inertia of such solids can be determined using experimental methods or numerical analysis.

Steiner's Theorem

If one wants to calculate the moment of inertia with respect to any axis that does not pass through the mass center of the mass, the Steiner's theorem becomes useful. It says that if the moment of inertia of the solid stiff with respect to the axis passing through its center of mass equals I_0, the moment of inertia of this mass rotating in relation to another axis parallel to the axis passing through its center of mass is:

$$I = I_0 + md^2, \tag{16.4}$$

where m is the mass of the body, and d is the distance between the axes. The above theorem is depicted in Figure 16.2.

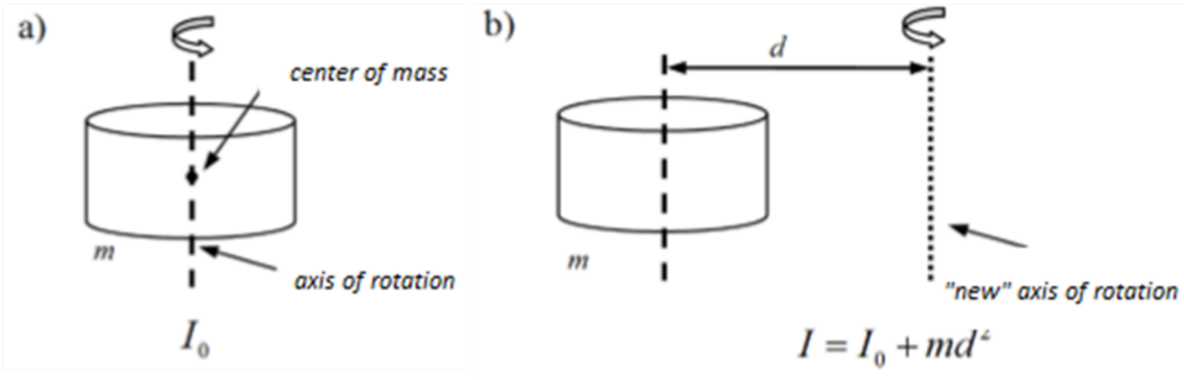


Figure 16.2. Illustration of Steiner's theorem: a) I_0 - moment of inertia of the solid with respect to the axis passing through its center of mass, b) I - moment of inertia of the solid relative to the "new" axis of rotation.

Measurement system

In the exercise, moments of inertia of the steel rod and disk will be determined. An additional task will be the experimental confirmation of Steiner's theorem. A torsion pendulum consisting of a stable base and a vertical axis mounted on bearings with very low friction will serve for testing. The axis and the base are connected by means of a spiral spring that allows torsional variations. At the end of the axis there is a bolt that allows the solids to be attached to it (Figure 16.3a). During the exercise, the pendulum axis will be fixed: a rod, a rod with two weights or a disk (Figure 16.3). There are notches on the rod, and special weights are screwed into the weights, which allows precise placing of weights on the rod (by moving the weight along the rod, we feel a clear jump of the screw to the cut). In a metal shield used to study the Steiner's theorem, a series of holes were drilled through which the target on the axis of the pendulum could be fastened. The torsion pendulum is a special case of a physical pendulum. If we assume that the swing of the pendulum is small (up to about 180°) and we neglect the resistance, its motion can be described as *simple harmonic movement*. In that case, the period T of the pendulum oscillation can be written as follows:

$$T = 2\pi\sqrt{\frac{I}{D}}, \tag{16.5}$$

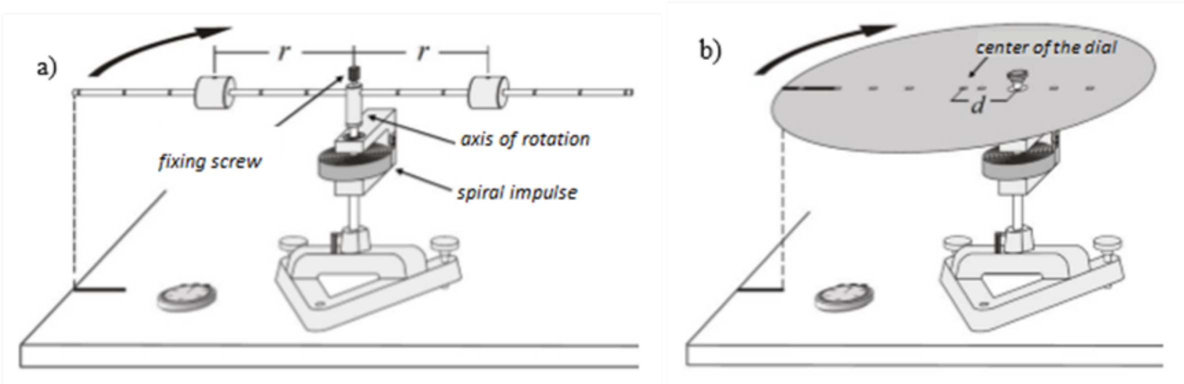


Figure 16.3. Torsion pendulum: a) a set for determining the restoring torque, b) a set for testing the Steiner's theorem

where I is the moment of inertia of the solid mounted on the axis of the pendulum, and D is steering torque pendulum springs. Steering torque D is a parameter characteristic for a given spring, depending on its construction, type of material, hardening method, etc.

Measurements and calculations

By having a torsion pendulum, we can calculate the moment of inertia of the solid attached to it. We will use for this purpose the transformed equation 16.5, the result of the vibration period measurement T and the designated restoring torque D:

$$I = D\left(\frac{T}{2\pi}\right)^2. \tag{16.6}$$

Determining the restoring torque

In the exercise in question, we will determine the spring driving moment by measuring the periods of fluctuation of a rod loaded with two weights (Figure 16.3a). Moment of inertia of the system consisting of a rod with two weights, located symmetrically in the distance r from the rotation axis, you can get the following equation:

$$I = I_P + 2m_C r^2, \tag{16.7}$$

where I_P is the moment of inertia of the rod, and m_C is the mass of each weight. Based on the formula 16.6, the moment of inertia of the rod mounted on the axis can be calculated using the equation:

$$I_P = D\left(\frac{T_P}{2\pi}\right)^2, \tag{16.8}$$

where T_P is the period of oscillation of the pendulum loaded with a rod (without weights). Substituting for the equation 16.6 formulas 16.7 and 16.8, we get the following relationship:

$$D\left(\frac{T}{2\pi}\right)^2 = 2m_C r^2 + D\left(\frac{T_P}{2\pi}\right)^2, \tag{16.9}$$

which after simplyfying takes the form:

$$T^2 = \frac{8\pi^2 m_C}{D} r^2 + T_P^2. \tag{16.10}$$

By making the following substitutions in the above equation, $y = T^2$, $x = r^2$, $a = 8\pi^2 m_C/D$ and $b = T_P$, the type dependency is obtained $y = a_{reg}x + b_{reg}$. It is a linear function, where the value a_{reg} is the directional coefficient of the line b_{reg} intersection with the axis y. Applying the linear regression method to the relation between the square of the vibration period to the square of the distance of weights from the axis: $T^2 = f(r^2)$, the slope coefficient of the straight line can be determined a_{reg}, and then the restoring torque:

$$D = \frac{8\pi^2 m_C}{a_{reg}}. \tag{16.11}$$

Determination of moments of inertia of a bar and disk in relation to their axis of symmetry

In order to determine the moment of inertia of a rod, you can use the previous period measurement for an unloaded rod and from equation 16.8. In order to determine the moment of inertia of the disk, one should fasten its center on the axis, and then measure the period of oscillation of the pendulum. Using the equation 16.6, we will determine the experimental value of the moment of inertia of the disk. If you want to compare the experimental values obtained with the theoretical values, you should weigh both blocks, measure the length of the rod and the diameter of the disk. The theoretical values of the moments of inertia are calculated from the following equations:

$$I = \frac{1}{12} ml^2, \tag{16.12}$$

$$I = \frac{1}{2} MR^2, \tag{16.13}$$

where m is the weight of the rod, l - length of the rod, M - disk mass, R - disk radius.

Experimental confirmation of Steiner's theorem

for testing, we will use a pendulum with a mounted disk in the configuration shown in (Figure 16.3b). The disk is then screwed on the axis for different distances d from the center of the disk (0, 2, 4, ..., 14 cm). For each position, we determine the period of oscillation of the pendulum, and then, using the equation 16.6, the moment of inertia of disk. In order to confirm Steiner's theorem, the theoretical moment of inertia of the disk based on the equation is calculated 16.4, which in the above case will take the following form:

$$I = \frac{1}{2} MR^2 + Md^2. \tag{16.14}$$

Experimental Procedure

A. Determination of restoring torque of spring

1. Determine: the mass of weights, the mass and length of the rod and the distance between the cuts on the rod. Save measuring accuracy.
2. Fasten the center of the rod to the pendulum axis, then tilt it about 90° and let go. Use the stopwatch to measure the time of five periods of deflection ($t = 5T$). Repeat this procedure twice more.
3. Slide the weights onto the rod and adjust them symmetrically so that their centers coincide with the notches on the rod closest to the center (with the precise setting feeling the screwing of the screw to the cut). Perform period measurements analogous to point 2.
4. Measurements from point 3 continue for the next distances r weights from the axis of rotation .
5. For each position of the weights, calculate the average time of five wobble periods, followed by the vibration period T.
6. Plot the relationship of the period to the square of the distance of the weights $T^2 = f(r^2)$.
7. Use the linear regression method to determine the slope coefficient of the straight line a_{reg} and its measurement uncertainty Δa_{reg}. Then, using the equation 16.11, calculate the restoring torque D and its measurement uncertainty. Complete the unit account.

B. Determination of moments of inertia of a bar and disk in relation to their axis of symmetry

1. Using the results obtained in points A.1 and A.2 and the equation 16.8, calculate the moment of inertia of the rod.
2. Determine the mass of the disk and its diameter. Mount the disk so that its center coincides with the axis of rotation. Perform measurements analogously to point **A 2.**
3. Calculate the average oscillation period of the disk and its moment of inertia (use equation 16.6).
4. Using the results of mass measurements of solids, rod length and disk diameter, calculate the theoretical values of moments of inertia based on formulas 16.12 and 16.13. Compare experimental and theoretical results.

C. Experimental confirmation of Steiner's theorem

1. Continue to measure vibration period analogically to the previous ones, changing the distance of the axis from axle of symmetry d every 2 cm (0, 2, 4, ..., 14 cm).
2. Calculate the mean values of the fluctuation periods and the moments of inertia of the disk using the equation 16.6.
3. Calculate from the equation 16.14 the theoretical values of the moment of inertia of the disk with respect to the subsequent rotational axes.
4. Compare in the table experimentally and theoretically obtained moments of inertia of the disk to confirm Steiner's theorem.

5. Using the common coordinate system, plot the experimental and theoretical moments of inertia of the disk as a function of the square of the axis distance from the center of the disk: $I = f(d^2)$.
6. Write down the final conclusions

Keywords:

- moment of inertia
- torsion spring
- Steiner's theorem
- linear regression

17. Determination of Young's modulus by the deflection method

Introduction

When a force perpendicular to its length acts on a longitudinal bar, it bends, and the value of the so-called *deflection arrows* S (Fig. 17.1) is proportional to the F force, and also depends on the geometrical dimensions, the method of fixing the bar and the type of material it is made of. This statement is *Hooke's law* of deflection.

Let us consider in more detail the deflection of a bar (beam), one end of which is fixed horizontally and the other end has a vertical force F (Fig. 17.1). Under the action of force, the upper layers of the bar are stretched, and the lower layers are compressed. There is a layer in the middle of the height, the length of which does not change. Perpendicular sections of a bar, when there is no load, they are parallel but form a certain angle φ when a force is applied. Figure 17.2 shows the considered sections through 1 and 2 and the angle φ between 1 and 2 ($1'$ is a parallel shift of the section to the line of intersection of the neutral layer N with section 2).

Consider a bar element of length Δr, thickness Δy and width b, which is x from the fixed edge and y above the middle layer. As a result of beam deflection, the test layer is elongated as if it were stretched by the force applied to the section with the area Δyb. According to *Hooke's law*, the elongation is proportional to the force and the initial length and inversely proportional to the cross-sectional area:

$$y\varphi = \frac{F_n \Delta x}{Eb\Delta y}, \tag{17.1}$$

where E - Young's modulus, F_n - tensile force of the considered elementary layer.

The same force, but in the opposite direction, acts on the elementary layer situated symmetrically below the neutral layer N.

The moment of force F_n with respect to the N layer

$$\Delta M = yF_n = E\frac{\varphi}{\Delta x}y^2 b\Delta y, \tag{17.2}$$

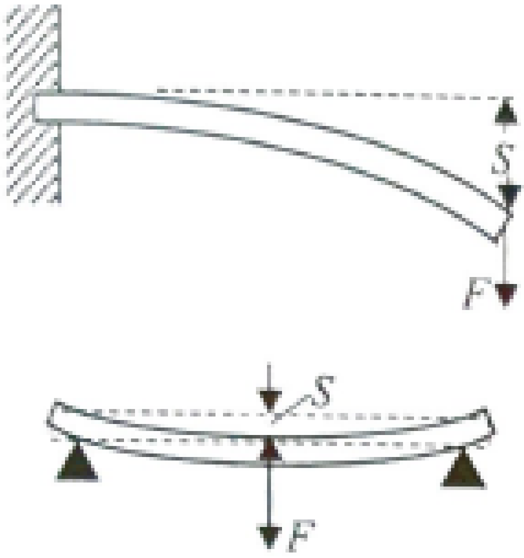


Figure 17.1. Deflection of bars

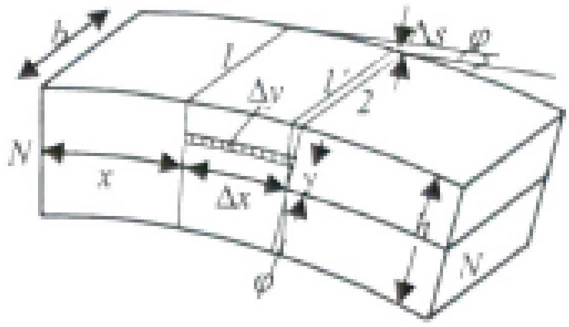


Figure 17.2. Element of a bending bar.

The total moment M of forces acting on all layers between sections 1 and 2 is calculated by integrating equation (17.2) with respect to y over the entire thickness

$$M = E\frac{\varphi}{\Delta x}\int_{-h/2}^{+h/2} y^2 b dy. \tag{17.3}$$

After marking

$$H = \int_{-h/2}^{+h/2} y^2 b dy \tag{17.4}$$

we can write the equation (17.3) in the form:

$$M = E\frac{\varphi}{\Delta x}H. \tag{17.5}$$

We obtained the above expression considering the deformation of the rod. The immediate cause of this deflection is the F force applied to the end of the bar. The moment of this force with respect to cross-section 2 is $F(l-(x+\Delta x))$ or after neglecting the value of Δx as small compared to x

$$M = (l-x)F. \tag{17.6}$$

The angle φ is also included between the tangents to the bar at the points where sections 1 and 2 intersect the top surface. Based on the drawing, we can write:

$$\varphi = \frac{\Delta S}{l-x}. \tag{17.7}$$

After taking into account the last relationship in equation (17.5) and comparing equations (17.5) and (17.6), we will obtain *an elementary deflection arrow*:

$$\Delta S = \frac{F}{EH}(l - x)^2 \Delta x. \tag{17.8}$$

We will obtain the *total deflection arrow* by summing up the analogous expression for all segments Δx. If we replace the segments Δx with infinitely small increments of dx, we can integrate equation (17.8):

$$\Delta S = \frac{F}{EH} \int_0^l (l - x)^2 dx. \tag{17.9}$$

After integrating, the expression for the integral deflection arrow takes the form:

$$S = \frac{F}{3EH} l^3. \tag{17.10}$$

The value of the H factor depends on the shapes and geometrical sizes of the bar. When the cross-section is a rectangle h high and b wide, the integration of equation (17.4) leads to the result:

$$H_{pr} = \frac{1}{12} bh^3. \tag{17.11}$$

Integrating a similar expression for a circular section gives

$$H_k = \frac{\pi}{4} r^4. \tag{17.12}$$

Substituting the values of H coefficients, we obtain the deflection arrows from the cross-sections, respectively

$$S_{pr} = \frac{4l^3}{Ebh^3} F, \tag{17.13}$$

$$S_k = \frac{4l^3}{3\pi E r^4} F. \tag{17.14}$$

The above formulas express Hooke's law with respect to deflection. The obtained formulas relate to a bar loaded with one side and one side fastened end. They can be easily adapted to the situation where the bar is resting freely at both ends and loaded in the middle. It then behaves as if it were fixed in the center with forces $F/2$ directed upwards at its ends. The force $F/2$ then acts on a $l/2$ bar.

After taking into account the above remarks in formulas (17.13) and (17.14), we will obtain formulas for deflection arrows of bars supported on both sides.

$$S'_{pr} = \frac{l^3}{4Ebh^3} F, \tag{17.15}$$

$$S'_k = \frac{l^3}{12\pi E r^4} F. \tag{17.16}$$

The formulas (17.15 and 17.16) are used to determine the Young's modulus because all the quantities appearing in them are easy to measure.

Measurements and calculations

The deflection arrow is measured with a catetometer - a device for remote height measurement - by setting the intersection of the spider's threads first on the edge of the unloaded rod, halfway along its length, and then on the same edge with gradually changing load on the rod. The deflection arrow is equal to the difference in the positions of the catetometer sight.

The length of l between the supporting edges is measured with a measure with a millimeter scale.

The width and height of the bar, or the diameter, are measured with a micrometer (if necessary, in many places). We change the load gradually by putting on weights of known masses.

Based on equations (17.15 and 17.16), we can see that the dependence of the deflection arrow on the applied force is a linear function, and the slope factor a of the line is equal to the expression at F and amounts to:

$$a_{pr} = \frac{l^3}{4Ebh^3}\text{(bar with rectangular section)}, \tag{17.17}$$

$$a_k = \frac{l^3}{12\pi Er^4}\text{(bar with circular section)}. \tag{17.18}$$

The slope factor can also be calculated by linear regression when only a series of forces and their corresponding deflection arrows are known. Thus, the Young's modulus remains the only unknown quantity in the last equations.

Measurements:

Note: In the experiment, the catetometer was replaced with an electronic micrometric sensor. The exercise aims at determining Young's modulus of solid rods/bars.

1. Determine width (b) and height (h) of the rod/bar (do the measurement at least three times).
2. Determine systematic error of the measurements.
3. Measure the distance l between the rod's supports.
4. Assume the mass of the weights $m = 50$ g with the systematic error of 1 g.
5. Align the rod on the supports so that the sensor points at its center.
6. Turn on the sensor and reset it to zero.
7. Increase the weight of the rod by adding weights. Each time measure the rod's deflection S and its systematic error.
8. Repeat the measurements while decreasing weight.
9. Repeat the experiment for the remaining rods/bars.

Report:

1. Calculate the force acting on the rod/bar $F_g = m \cdot g$ where $g = 9.81$ m/s^2.
2. For each rod/bar, plot the deflection as a function of the force: $S = f(F_g)$.
3. Determine the slope coefficients (linear regression a_{reg}).
4. Calculate the Young modulus E from the relation: $a_{reg} = \dfrac{l^3}{4Ebh^3}$
5. Calculate its uncertainty (ΔE).
6. Present the final results of the experiment (properly rounded) and compare to the table values (literature values) for the tested materials.
7. Write down the final conclusions

Keywords:

- Interatomic interaction: the relationship of potential energy and force from the distance
- Elastic and plastic deformation, limit of proportionality, elasticity, endurance
- Relative deformation, normal and tangential stresses
- Hooke's law, Young's modulus
- Deflection: deflection arrow, mathematical description
- Calculation of Young's modulus on the basis of performed measurements
- Linear Regression

18. Investigation of the uniformly accelerated motion using a computer measuring set

Introduction

The type of movement the body performs depends on the property of the force causing it. For example, central force causes circular motion, force proportional to deflection from equilibrium - harmonic motion, zero force - uniform motion, constant force - uniformly varying motion. The position of a material point in space is described by a guide vector, expressed by three coordinates

$$\vec{r} = \vec{r}(x, y, z). \tag{18.1}$$

The coordinates of the moving point vary with time, and the tip of the guidance vector moves to trace the path along which the movement takes place. In general, the path may be a curved line and its increments in successive time periods may be different. The motion is characterized by three parameters: s path, v velocity and a acceleration. In general, all three quantities are vectors, but for the purposes of this exercise, we will only consider straight-line motion and scalar quantities. If we take the path as the starting point, the remaining quantities are defined as follows:

$$v = \frac{ds}{dt}, \tag{18.2}$$

$$a = \frac{d^2s}{dt^2} \quad \text{or} \quad a = \frac{dv}{dt}. \tag{18.3}$$

Speed and acceleration are generally quantities that vary with time. Derivatives appearing in the above equations allow to calculate the instantaneous speed and instantaneous acceleration at any time t, because the increments used in the calculations are very small.

In the case of a uniformly varying motion, the acceleration is constant and from equation (21.3) we can determine the speed at any moment. Suppose at time $t = 0$ the velocity is Vo, and at any time t it is v. After transforming the equation to the form $dv = adt$ we can integrate both sides within the appropriate limits:

$$\int_{v_0}^{v} dv = a \int_{0}^{t} dt. \tag{18.4}$$

Computing the integrals leads to the dependence of speed on time

$$v = v_0 + at. \tag{18.5}$$

In a similar way we can obtain integrals containing the path:

$$\int_{0}^{s} ds = \int_{0}^{t} (v_0 + at)dt. \tag{18.6}$$

and ultimately the dependence of the path on time

$$s = v_0 t + \frac{at^2}{2}. \tag{18.7}$$

Earth acceleration

Every body in the Earth's gravitational field is exerted by a force towards the center of the Earth. This force is called the force of gravity, and its value is determined by the law of universal gravity:

$$F = G\frac{mM}{R^2}. \tag{18.8}$$

where G is the gravitational constant, m - the mass of the body, M - the mass of the Earth, R - the distance from the center of the Earth.

The same force can be expressed by the second law of dynamics:

$$F = mg. \tag{18.9}$$

The value g in this equation is the acceleration due to gravity. Comparing the last two expressions, we see that the gravitational acceleration can be expressed in the form of the equation:

$$g = G\frac{M}{R^2}, \tag{18.10}$$

which shows that it is not constant, but changes with the distance from the center of the Earth. However, for phenomena occurring in the range of small heights ΔR above the Earth's surface, it can be assumed with sufficient accuracy that the distance from the center of the Earth is constant, so the acceleration of gravity is in this fixed range.

EXAMPLE

Calculate the change in the acceleration of gravity at the transition from the Earth's surface to $\Delta R = 6.37$ km. The radius of the Earth is approximately 6370 km, so $\Delta R/R = 0.001$; we can apply the error theory methods to such a small change. Applying the logarithmic differential to equation (21.10), we get $\Delta g/g = 2\Delta R/R$. The numeric value for $\Delta g/g$, the relative change in acceleration, is 0.2 %.

Inclined plane

The force of gravity mg of a body resting on an inclined plane is divided into two components: the parallel component F and the component N perpendicular to it. The effect of the force F is movement along the plane and therefore it is called the sliding force. The N component does not lead to movement, but causes pressure of the body on the ground, hence its name contact force. Both forces depend on the angle of inclination of the plane and their values are respectively (see Fig. 18.1):

$$F = mg\sin\alpha, \tag{18.11}$$

$$N = mg\cos\alpha. \tag{18.12}$$

Pressure on the ground results in a frictional force that makes it difficult for the body to move or slows down movement. In any case, the friction force is proportional to the pressure:

$$T = \mu N = \mu mg\cos\alpha. \tag{18.13}$$

where μ is the coefficient of friction depending on the type of body surface and ground. In the range of low speeds, the μ value is constant.

Measurements and calculations

In the exercise we examine two cases of uniformly accelerated motion:

- free fall - to determine the acceleration due to gravity,
- movement on an inclined plane - to determine the coefficient of friction. In both cases we use the computer measuring set described in chapter 12 in a text book [2].

Free fall

In the exercise, we observe the free fall of the ladder shown in fig. 18.2. As a sensor, we use a photo frame that directly measures the time of passing subsequent crossbars. Before starting measurements, the parameter - distance between adjacent bars d must be entered into the program. The parameter is entered into the window opened by clicking on the sensor icon in the settings window. On the basis of the measured times,

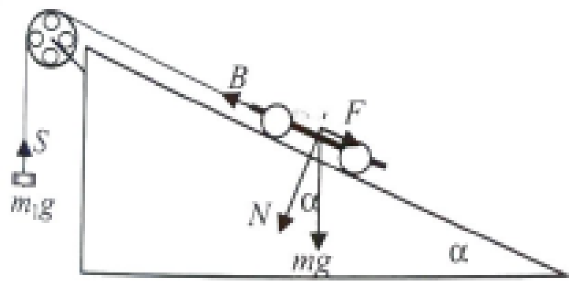


Figure 18.1. Forces acting on an inclined plane

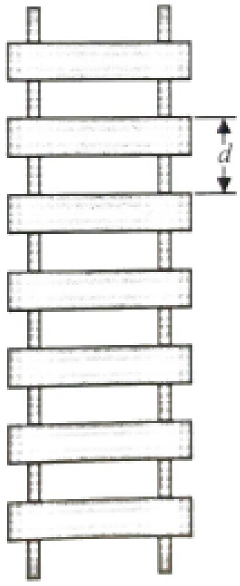


Figure 18.2. Ladder for determining the acceleration of gravity

the program can calculate the distance traveled, the speed between successive bars, as well as acceleration on individual sections. The method of selection and the form of data presentation are described in chapter 3, section 12 (chapter 12 [2]). In this exercise, we present the results as graphs of road and speed (two graphs) against time. Windows containing these graphs should be opened before starting the measurements. Also, in the speed plot, turn on *Statistics* and select *Curve fit* and *linear function*. We treat the road chart qualitatively. It serves us only to visually check whether the corresponding relationship is a polynomial of the second order, according to equation (18.7). The velocity graph will be used to determine the acceleration due to gravity. In the case of free fall, in equation (18.5) we convert the general acceleration to the acceleration due to gravity g

$$v = v_0 + gt. \tag{18.14}$$

It can be seen from the above equation that velocity depends linearly on time, and g is the slope coefficient of the line describing the relationship $v(t)$. We find the value of g from linear regression.

In the statistics field we see the linear equation $y = a1 + a2x$ and the values of a_1 and a_2 calculated by the linear regression method. Regarding our chart, $a_1 = v_0$ and $a_2 = g$. Check that the measurement points are around a straight line. If not, select the appropriate range of measurements in which this condition will be met.

Movement on an inclined plane

To determine the friction coefficient, we use the inclined plane shown in fig. 18.1

and the measuring computer system with a sensor in the form of a photo frame with a disc.

The sliding force F and the force B directed in the opposite direction acts on the trolley with the mass m and is the sum of the friction force T and the thread tension S. The second law of dynamics for a wheelchair will then take the form:

$$ma = F - T - S, \tag{18.15}$$

and for the counterweight m_c

$$m_c a = S - m_c g. \tag{18.16}$$

From the last equation we calculate S and insert into the formula (18.15) together with the previously calculated expressions for T and F. We get the equation:

$$ma = mg\sin\alpha - \mu mg\cos\alpha - m_c(a+g), \tag{18.17}$$

from which we finally calculate the friction coefficient:

$$\mu = \frac{m(g\sin\alpha - a) - m_c(a+g)}{mg\cos\alpha}. \tag{18.18}$$

The above equation shows that the calculation of μ requires the knowledge of the masses of the trolley and counterweight, the slope angle and the acceleration of the trolley. We determine the masses using a scale, and the angle of inclination - from measurements of the height and length of the slope. We use a computer set to determine the acceleration.

The counterweight is connected to the cart by means of a thread thrown through a pulley located between the arms of the photo frame. The rotating disk generates voltage pulses reflecting the obstruction or exposure of the photo frame detector through the holes in the disk. The measuring system directly measures the rotation time between adjacent holes. After entering the arc length corresponding to the rotation between adjacent holes, the program can calculate the distance traveled by the trolley and the speed or acceleration in the following points. This parameter is entered into the window opened by clicking on the sensor icon in the settings window.

Motion on an inclined plane is uniformly accelerated motion; the speed of the trolley is given by equation (21.5). This is a linear equation for $v(t)$ with the slope factor of a. After enabling *Statistics* in the Speed Graph pane, select *Curve Fit* and *Linear Function*. The statistics field shows the equation $y = a_1 + a_2 x$ and the values a_1 and a_2 calculated by linear regression. Regarding our chart, $a_1 = v_0$ and $a_2 = a$. Check that the measuring points are in a straight line. If not, an appropriate range of measurements should be selected in which this condition will be met.

Thus, we find the acceleration value. After inserting all non-measured values into equation (18.18), we calculate the friction coefficient. Measurements and calculations can be repeated for different slope angles and for different counterweight loads.

Measurements:

Start the measuring system.

Turn on the computer and start the Science Workshop program. To read more about the use of a computer in laboratory measurements please **see section 12**.

A. Determination of gravitational acceleration:

1. (see 12)
2. Open the appropriately configured file (*drabinka.dat*).
3. When releasing the ladder through a photo gate, plot the speed (v) versus time (t) in the program.
4. Read, from the statistics field in the program, the slope factor of the straight line $v = v_0 + gt$ we get linear fit $y = a_1 + a_2x$ - acceleration ($g = a_2$) of the ladder movement. Repeat the measurement from ten to fifteen times.

B. Determination of the friction coefficient of teflon-teflon:

1. Open the appropriately configured file (*rownia.dat*).
2. The Teflon block can be additionally loaded with weights. For each measurement, the weight of the loaded block (m), the mass of the counterweight (m_c) and the angle of inclination (α) should be recorded.
3. Release the block downwards. Read the slope factor of the straight line - acceleration (a_x) of the body movement ($v = v_0 + a_xt$, $a_2 = a_x$).
4. Perform ten measurements for different combinations of block weight, counterweight weight, and pitch angle.

Additional information:
The weight of the empty Teflon block used in the exercise: 499 g.
Weight bar for hanging weights: 9.5 g.

Report:

A. Determination of gravitational acceleration:

1. Calculate the mean acceleration due to gravity (g) and its uncertainty (Δg).
2. Present the final results of the experiment (properly rounded).

B. Determination of the friction coefficient of teflon-teflon:

1. Calculate the coefficient of friction (μ_x) equally for each combination of motion-related parameters.
2. Calculate the average value of the friction coefficient ($\mu = \dfrac{m(g\sin\alpha - a) - m_c(a+g)}{mg\cos\alpha}$) and its uncertainty ($\Delta\mu$).
3. Present the final results of the experiment (properly rounded).
4. Write down the final conclusions

Keywords:

- The type of motion and the acting force, definitions of speed and acceleration
- Uniformly variable motion: speed and distance dependence on time
- Law of universal gravity, acceleration of gravity as a function of distance from the center of the Earth
- Shearing force, pressing force and friction force on an incline
- II dynamics for the movement of a counterbalanced truck

- Measurement program (see chapter 3, section 12 or chapter 12 in [2]), settings window, opening the chart window
- Sensor assignment, graph selection
- *Statistics*: switching on and off, setting options; matching functions, interpretation of matching parameters

19. Determination of the dependence of the viscosity coefficient on temperature

Introduction

Particles in a moving fluid (liquid or gas) have different velocities. For example, in a pipe, the speed of the particles directly touching the wall is zero, and the particles moving along the axis of the pipe have the highest velocity. We can mentally divide any fluid into layers in such a way that all particles in a single layer have the same velocity. For a pipe, such a layer would be in the shape of a thin cylinder. In general, the shape of the layer is determined by the shape of the vessel in which the flow takes place.

In all real fluids, friction forces occur between the layers. From the side of the faster moving layer, a accelerating force acts on the slower moving layer. On the other hand, from the side of the slower moving layer, a braking force acts on the faster moving layer (Fig. 19.1). These forces, called internal friction forces, are directed tangentially to the surface of the layers. The internal friction force (F_T) is greater, the greater the surface area (S) and the greater the velocity gradient in the direction perpendicular to the motion (dv/dz).

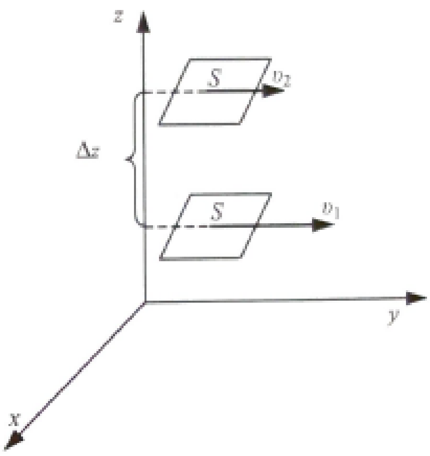


Figure 19.1. Movement of selected layers of viscous fluid

$$F_T = \eta S \frac{dv}{dz}. \tag{19.1}$$

The velocity gradient is the limiting value of the ratio $(v_1 - v_2)/\Delta z$ for $z \to 0$, numerically equal to the difference in velocity of distant layers by a unit length. The quantity η, depending on the type of liquid, is called *the internal friction coefficient* or *the viscosity coefficient*. The dimension of the viscosity index is kg/(ms). A liquid has a unit viscosity if the force of 1 N acting on an area of 1 m^2 causes a velocity decrease of 1 m/s over a distance of $z = 1$ m. In the less frequently used CGS system, a unit with the dimension g/(cm·s), called *poise*, is used. Both units are easy to compare: 1 kg/(m·s) = 10 poise.

The viscosity of the liquid is highly dependent on temperature; its value decreases with increasing temperature. Temperature changes in gas viscosity are opposite - gas viscosity increases with temperature.

An interesting phenomenon was discovered by Kapica. He found that liquid helium at $-271°C$ (2 K) becomes *overfluid* in that its viscosity is zero. A solid body moving in a non-viscous liquid does not meet any resistance, while the resistance given to the body by a viscous liquid causes that its movement under the action of a constant force is uniform (except for the initial section) and not accelerated.

The movement of a ball in a viscous liquid was described by Stokes - in our exercise I will use the properties of this movement to determine the viscosity coefficient.

Principle of measurement

At low ball velocities, when no vortices are formed, the drag force is directly determined by the viscosity of the liquid. The liquid layer directly adjacent to the ball sticks to its surface and is completely lifted by it. This layer carries the next layer with it, but its speed is slower. The speed of subsequent layers is even lower as a result of internal frictional forces.

According to *Stokes's law*, the force of internal friction is directly proportional to the speed and radius of the ball, and is expressed by the formula:

$$F_T = 6\pi\eta r v, \tag{19.2}$$

where: r - radius, v- ball speed.

In the exercise, the ball falls in the liquid under the influence of gravity:

$$F_G = mg = \frac{4}{3}\pi r^3 d_k g, \tag{19.3}$$

where d_k is the ball density, g - gravitational acceleration.

Apart from the above-mentioned forces, an important role is played by the buoyancy which, according to Archimedes' law, is expressed by the formula

$$F_W = mg = \frac{4}{3}\pi r^3 d_c g, \tag{19.4}$$

where d_c- liquid density.

Taking into account the directions of action of the forces, we will write the resultant force as:

$$F = F_G - F_W - F_T. \tag{19.5}$$

From the moment the ball is released ($v_0 = 0$), its movement is accelerated because the force of gravity is greater than the sum of the buoyancy and viscosity forces. As the speed increases, the force F_T also grows, which leads to a decrease in the resultant force F. This continues until the resultant force becomes zero. From this point on, the ball's movement becomes uniform.

In the conditions of the experiment we determine the speed by measuring the time t in which the ball travels the fixed path l.

The equations (19.5) for the uniform motion range ($F = 0$) we can determine the viscosity coefficient. After expressing the appropriate forces by equations from (19.5) to (19.4) and transforming equation (19.5), we obtain the expression for the viscosity coefficient:

$$\eta = \frac{2(d_k - d_c)gr^2t}{9l}. \tag{19.6}$$

This equation is the basis of the viscosity index method described below.

The Höpler viscometer

The purpose of this exercise is to find the value of the viscosity coefficient with temperature. We will use a Höpler viscometer and an ultrathermostat for this purpose. In the Höpler viscometer, the structure of which is shown in Fig. 19.2, the diameter of the cylinder only slightly exceeds the diameter of the ball, and the cylinder itself is slightly oblique, thanks to which the ball rolls on the cylinder wall in a uniform motion. Formula (19.6) is also used in this case, but we will write it now as:

$$\eta = K(d_k - d_c)t, \tag{19.7}$$

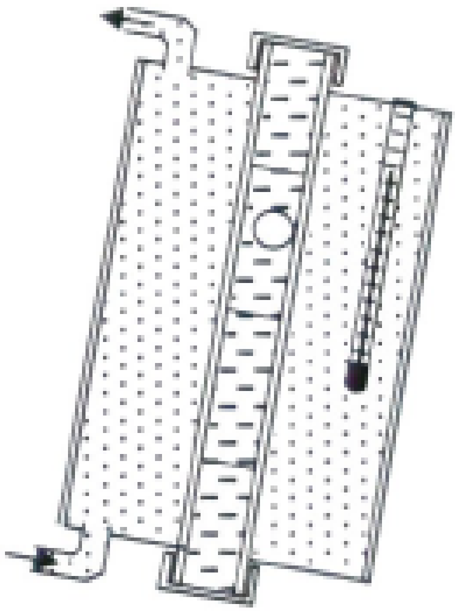


Figure 19.2. The Höpler viscometer

where K is the constant of the instrument, experimentally determined by measurement for liquids with a known viscosity index.

The ball and liquid density is based on the tables. The only thing that needs to be measured is the time the ball falls between the two levels marked on the cylinder. In some types of instruments, three levels are marked, which makes it possible to determine the uniformity of the ball movement. When the movement is uniform, the times of covering the upper and lower section are equal. The measurement of the ball falling time can be repeated many times - it is enough to turn the device through an angle of 180° each time

The viscometer cylinder is surrounded by a temperature-controlled water jacket, the value of which is indicated by a thermometer immersed in water. The casing of the water jacket is connected with flexible wires to an ultrathermostat, in which the water temperature is regulated. As a result of heat losses in the conduits supplying the thermostating fluid to the viscometer, its temperature differs from the value set on the ultrathermostat controller (for details see section 7). We take the temperature indicated by the viscometer thermometer into account.

To find the constant K we use equation (19.7). It is substituted by the value η from the tables for 20°C and t - the falling time measured for the temperature of 20°C. The value of n for other temperature values is calculated directly from equation (19.7) by substituting the already known constant K and the measured fall time.

Measurements:

1. Set the thermostat control to 20°C or 25°C. Wait for the system temperature to stabilize.
2. Read the temperature (T) of the viscometer from the thermometer inside it. Record the accuracy of the temperature measurement (ΔT).
3. Measure the ball drop time twice ($t_{20^\circ\mathrm{C}}$ or $t_{25^\circ\mathrm{C}}$). Record the accuracy of the time measurement (Δt).
4. Double-measure the ball dropping times for successive temperature values. Change it approximately every 3°C up to 40°C.

Report:

1. Calculate the viscosimeter constant K from the relation: $\eta = K(d_b - d_g)t$, where d_b is the density of the ball equal to (8150 ± 10) kg/m^3, $d_g = 1262,01$ kg/m^3 [8], is the density of the liquid (glicerine), $\eta_{20^\circ\mathrm{C}} = 1.499$ Pa·s at 20°C, $\eta_{25^\circ\mathrm{C}} = 0.945$ Pa·s at 25°C [8].
2. Calculate the coefficient of viscosity η for each temperature.
3. Plot the coefficient of viscosity as a function of temperature. Plot the error rectangles (ΔT, $\Delta\eta$)
4. Write down the final conclusions

Keywords:

- Internal fluid friction, friction force and velocity gradient, viscosity coefficient, viscosity coefficient units, temperature dependence
- Stokes's law, buoyancy, resultant force acting on a ball, condition of uniform ball motion in a fluid
- Höpler viscometer, calculation of the constant K, calculation of the viscosity coefficient at any temperature

20. Determination of the stiffness modulus using the dynamic method

Torsional deformation

Forces acting tangentially to the surface of a solid body cause whip shifts of individual elements and lead to shear or twist deformations. In a deformed body, there is an imbalance between atomic forces, and as a result, elastic resistance forces appear. The ratio of the tangential force F_s, to the surface S it acts on, is called the tangential stress τ

$$\tau = \frac{F_s}{S}, \tag{20.1}$$

Hooke's law, according to which the stress is proportional to the strain, in the case of tangential stresses takes the form:

$$\tau = G\varphi, \tag{20.2}$$

where w is the measure of angular deformation, and G - the stiffness modulus of the dimension $\mathrm{Nm}^{-2}\cdot\ \mathrm{rad}^{-1}$.

As an example of a torsional deformation we consider a thin-walled cylinder subjected to tangential forces applied to the upper base as shown in Fig. 20.1. The lower base is fixed in place. The twist angle o can be expressed by rotating the upper base α.

$$\varphi\frac{s}{l} = \frac{r}{l}\alpha \tag{20.3}$$

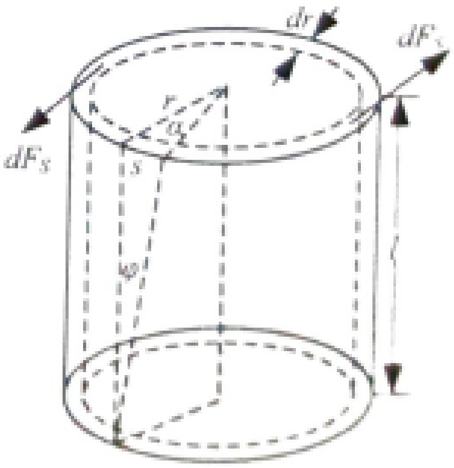


Figure 20.1. Torsion of a thin-walled cylinder

and the tangential stress, as defined in (20.1), in the form

$$\tau = \frac{dF_s}{2\pi r dr}. \tag{20.4}$$

After taking into account expressions (20.3) and (20.4) in equation (20.2), we find the tangential force dF_s:

$$dF_s = \frac{2\pi G\alpha r^2}{l}dr. \tag{20.5}$$

and after multiplying the above equation by r on both sides, we get the moment of force dM:

$$dM = \frac{2\pi G\alpha r^3}{l}dr. \tag{20.6}$$

Under the conditions of static equilibrium, the equation (20.6) can be read in two ways - taking the moment of force as the cause and the deformation as the effect, or vice versa. In the first case, the torsion by the angle α is the result of an external force with the moment dM, while in the second - the moment of elastic force is the result of the existing deformation.

The cylinder shown in Fig. 20.1 can be treated as an element of a full cylinder. The moment of force twisting the cylinder base by the angle α is obtained by integrating the contributions from the rings with radii from $r = 0$ to $r = R$.

$$M = \frac{2\pi G\alpha}{l}\int_0^R r^3 dr. \tag{20.7}$$

After integrating, we get the expression for the moment of force

$$M = \frac{\pi G r^4}{2l}\alpha. \tag{20.8}$$

in which, apart from the stiffness modulus G, there are quantities readily available for measurement, so equation (20.8) can be used to determine the stiffness modulus.

Dynamic method

To determine the stiffness modulus, the static (for thick bars) and dynamic (for thin bars and wires) methods are used. In the dynamic method, the tested wire is attached with its upper end to a fixed holder, and a vibrator is suspended at the lower end (Fig. 20.2). The vibrator consists of rods equipped with pins that enable the imposition of additional loads.

When the vibrator is twisted an angle, there will be a moment of elastic forces in the wire trying to restore equilibrium. When the vibrator is released it will vibrate. Forces and moments of forces occurring in a stranded bar or wire have been considered in detail in the previous section.

Equation (20.8) shows that the moment of force is proportional to the deflection angle, which is the basic property of harmonic motion. Thus, the motion of the vibrator is harmonic motion and the period of this motion

$$T = 2\pi\sqrt{\frac{I}{D}}, \tag{20.9}$$

where: I - moment of inertia, D - steering moment (see chapter 14).

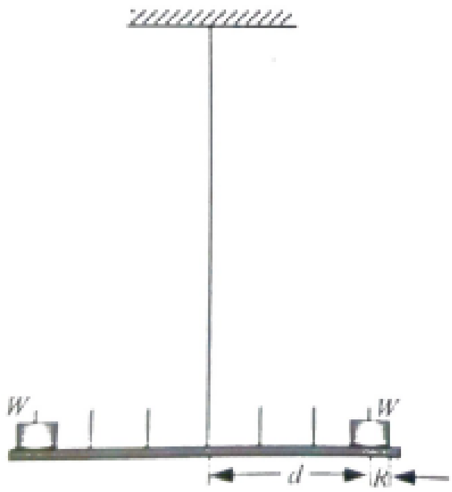


Figure 20.2. Device for determining the stiffness modulus by the dynamic method

The driving moment is calculated from equation (20.8), taking into account that $D = M/\alpha$

$$D = \frac{\pi G r^4}{2l}, \tag{20.10}$$

The moment of inertia of an unloaded vibrator is usually difficult to calculate directly and therefore we will use a method to eliminate this quantity. After placing additional rollers on the vibrator arms, the moment of inertia will increase by I_1, and the vibration period will increase

$$T_1 = 2\pi\sqrt{\frac{I + I_1}{D}}, \tag{20.11}$$

Equations (20.9) and (20.11) allow you to eliminate I I by squaring both equations and then subtracting them both sides. After these operations, only the transformation that leads to the expression of the driving moment in the form:

$$D = \frac{4\pi^2 I_1}{T_1^2 - T^2}. \tag{20.12}$$

Comparing equations (20.10) and (20.12) with each other, we can find the torsional modulus:

$$G = \frac{8\pi l I_1}{r^4(T_1^2 - T^2)}, \tag{20.13}$$

where: l - wire length, r - wire radius, T - vibration period of the unloaded or preloaded vibrator, T_1 - vibration period of the vibrator loaded with known masses. The last equation is the basis of the method of determining the stiffness modulus and indicates the quantities that we need to measure in order to calculate this modulus.

Additional moment of inertia I_1 is obtained by placing cylinders of known mass on the appropriate pins of the vibrator. If the distance of the roll axis from the vibrator is d, the number of rollers N, and the mass of each m, then according to Steiner the moment of inertia of these rollers is expressed by the formula:

$$I_1 = NI_0 + Nmd^2, \tag{20.14}$$

where I_0 is the moment of inertia of a single cylinder about its axis of symmetry. For a cylinder with radius R and weight m: $I_0 = (1/2)mR^2$.

Measurements:

1. Measure the length (l) of the wire to be tested. Note the measurement accuracy (Δl).
2. Measure the wire diameter (ϕ) several times. Specify the measurement accuracy ($\Delta\phi$).
3. Take dimension of the vibrator to determine the distance of the pins from the axis of rotation (d_1, d_2, d_3).
4. Measure the diameters of subsequent rollers loading the vibrator ($\phi_w \to R = \phi_w/2$). Note the measurement accuracy ($\Delta\phi_w$).
5. Pre-load the vibrator (e.g. with four weights). Measure the duration (t) of the ten oscillations of the vibrator so loaded. Note the accuracy of the time measurement (Δt).
6. Change the moment of inertia of the vibrator by adding subsequent loads. Each time measure the duration of ten oscillations of the vibrator with added loads (t_d).
7. Take measurements for at least six mass distributions, taking care not to remove the weights preloading the vibrator.

Report:

1. Calculate the average diameter of the tested wire (ϕ) and its accuracy ($\Delta\phi$).
2. Determine the distance (d_1, d_2, d_3) of the axis of the vibrator pins from its center.
3. Using the Steiner's theorem, calculate by what value (I_d) the moment of inertia of the vibrator changes in the case of subsequent loads relative to the moment of inertia of the preloaded vibrator. $I = NI_0 + Nmd_i^2$, where I_0 is moment of inertia of single weight with respect to its axis of symmetry, for cylinder of radius R and mass m: $I_0 = (1/2)mR^2$, N - are the number of weights, d_i is distance of weight axis to axis of vibrator, (where $i = 1, 2, 3$).
4. Calculate the stiffness modulus for each mass distribution (G_d). $G_d = \dfrac{8\pi l I_d}{r^4(T_1^2 - T^2)}$, where l - length of vibrator wire, r- radius of wire, T - period of vibrations for empty or pre-loaded vibrator, T_1 - period of vibrations for loaded vibrator.

Please **note** that $I_d = I_n - I_{pre}$, where I_n is moment of inertia for n-th load of vibrator (usually is necceasery to make n from 6 to 10 diffrent loads of vibrator) and I_{pre} is the moment of inertia of preloaded vibrator.

5. Calculate the average stiffness modulus (G) and its uncertainty (ΔG).
6. Present the final result (appropriately rounded stiffness modulus and its uncertainty).
7. Compare the obtained result to the table value for steel
8. Write down the final conclusions

Keywords:

- deformations dependence on stress, normal and tangential stresses,
- deformations depending on the type of stress,
- Hooke's law, stiffness (torsion) module,
- cylinder and roller torsion,
- harmonic motion: moment of force and deflection, period.

5. Electromagnetism

21. Determining the capacitance of a capacitor by means of relaxation vibrations.

Introduction

A capacitor is a system of two metal plates of any shape separated by a dielectric. In the state of charge on each of the plates there is an electric charge Q of the opposite sign, and there is a potential difference (voltage) U between the plates. The *capacitance* of a capacitor is the ratio of charge to voltage

$$C = \frac{Q}{U}. \tag{21.1}$$

The capacitance of a capacitor depends on its geometry, i.e. the shape, size and distance of the plates, as well as the type of dielectric between them. The capacity of capacitors with a sufficiently symmetrical structure (e.g. flat, cylindrical, spherical) is described by simple formulas given in basic physics textbooks.

The capacitor is charged by connecting a source with electromotive force (EMF) ε to a circuit with series-connected resistance R and capacitance C (in Fig. 21.1, switch position a), while discharging - by disconnecting the source from the circuit (switch position b).

Charge process

At any time of charging, there is a charge q on the covers, and a current i in the circuit. According to the Kirchhoff 2^{nd} law, the potential drops on the capacitor and on the resistor are compensated by the EMF

$$\varepsilon = iR + \frac{q}{C}. \tag{21.2}$$

After differentiating this equation and taking into account the relation $i = dg/dt$, we get

$$\frac{di}{dt} + \frac{1}{RC}i = 0. \tag{21.3}$$

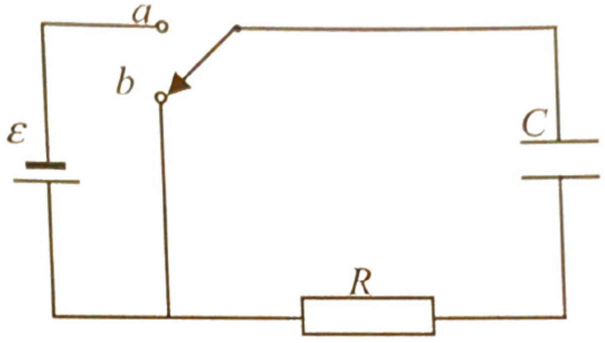


Figure 21.1. The RC circuit

It is a differential equation in which we can separate the variables and then integrate both sides of the equation. As a result of this procedure, we will obtain a solution in the form of:

$$i = i_0 e^{\frac{t}{RC}} = \frac{1}{RC} e^{\frac{t}{RC}}, \tag{21.4}$$

where i_o is the integration constant defined by the initial conditions. At the initial moment of charging ($t = 0$) the charge on the capacitor plates is equal to zero and from equation (21.2) it follows that then the current at $i_o = \varepsilon/R$.

The voltage on the capacitor U_c at any moment is $\varepsilon - Ri$ and changes with time according to the equation:

$$U_c = \varepsilon(1 - e^{\frac{t}{RC}}). \tag{21.5}$$

After a sufficiently long time, the capacitor is fully charged. Mathematically we find that $U_c \to \varepsilon$ when $t \to \infty$. In practice, we consider the capacitor charged after $t = 5RC$.

Discharge process

When the plates of the charged capacitor are connected directly with the resistor R (switch in position b), the current will flow through the resistor in the opposite direction than when charging. In this case, the second law of Kirchhoff takes the form:

$$Ri + \frac{q}{C} = 0. \tag{21.6}$$

After considering again that $i = dq/dt$, we get the differential equation

$$R\frac{dq}{dt} + \frac{q}{C} = 0. \tag{21.7}$$

The solution to this equation is the function:

$$q = q_0 e^{\frac{t}{RC}}, \tag{21.8}$$

where q_0 is the initial charge on the capacitor - this is the charge of the charged capacitor: $q_0 = C\varepsilon$.

We find the intensity of the current during discharge by differentiating the equation (21.8)

$$i = -\frac{\varepsilon}{R} e^{\frac{t}{RC}}. \tag{21.9}$$

Dividing equation (21.8) by C, we find the voltage across the capacitor at any time of discharge:

$$U_c = \varepsilon e^{\frac{t}{RC}}. \tag{21.10}$$

In the equations describing the charging and discharging of a capacitor, there is a quantity RC having a time dimension. This quantity is called the *circuit time constant* and determines the speed of both charging and discharging the capacitor. After the time $t = RC$ from the moment the expression $\exp(-t/(RC))$ is started loading or unloading is $1/e$ ($e = 2.71828$). The corresponding equations given above show that the charging or discharging current and the discharge voltage with the RC time decrease e-fold compared to the initial value, while during charging, the voltage on the capacitor after this time reaches $(1 - 1/e)$ values fully charged.

Relaxation vibrations

If we connect a neon lamp to the RC circuit in parallel to the capacitor (Fig. 21.2), periodic, asymmetrical voltage increases and decreases on the capacitor will occur, called *relaxation vibrations*.

A neon lamp, also known as a stabilivolt, is a glass bulb filled with neon gas, under a pressure of about 20 mm Hg, containing two metal electrodes with low work output, e.g. barium. If a small voltage is applied to the neon tube, the current does not flow through it due to the low conductivity of the gas. When the value of U_z (ignition voltage) is exceeded, the gas is ionized, its glow is visible, a current flows through the neon lamp and the voltage on the capacitor decreases. The started avalanche ionization continues at slightly lower voltages - it stops only when the voltage drops below the value known as the extinction voltage U_g.

We use the described properties of the neon lamp to obtain relaxation vibrations (21.2). The capacitor C charges from the DC source through the resistance R. The voltage on the capacitor plates increases exponentially, according to equation (21.5). When the voltage reaches U_z, the N neon lamp lights up. Since the resistance of the lightening neon lamp is very small, the capacitor (described by equation 21.10) is quickly discharged to the voltage U_g. After the neon lamp turns off, the capacitor is charged again and then discharged. The described processes repeat cyclically.

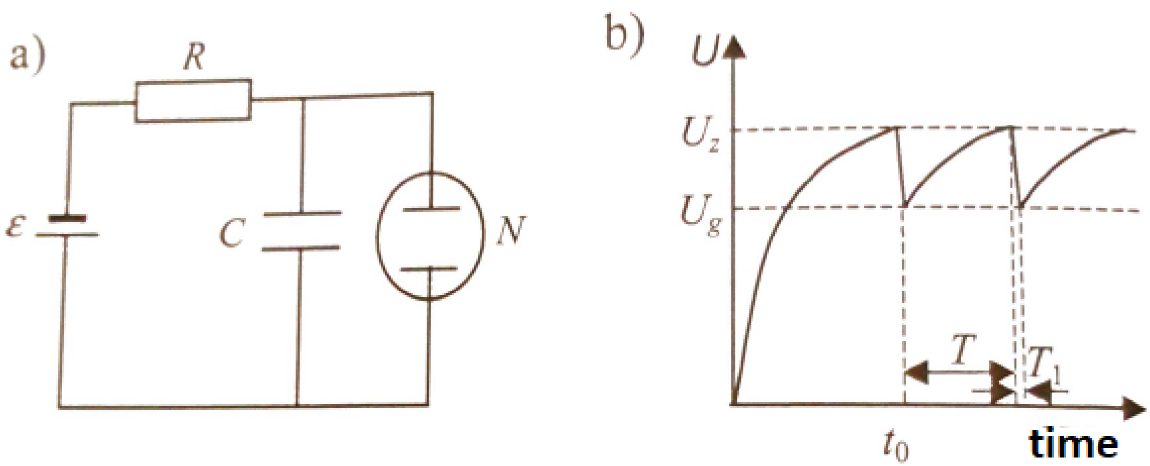


Figure 21.2. The circuit for generating relaxation vibrations (a) and the voltage waveform on the capacitor (b)

A single period consists of two parts: the voltage rise time T, determined by the capacitance C and the resistance R, and the time T_1, in which the voltage across the capacitor decreases, determined by the same capacitance and resistance of the neon tube. Due to the fact that the resistance of the glowing neon lamp is much lower than the resistance R, the discharge time is a small fraction of the entire period and in most cases we can assume that the relaxation vibration period is equal to the capacitor charging time from the extinguishing voltage U_g to the voltage ignition U_z.

In the first charging cycle, the U voltage will be reached after t_0. Equation (21.5) for the time t_0 takes the form:

$$U_g = \varepsilon\left(1 - e^{\frac{t_0}{RC}}\right). \tag{21.11}$$

We can write an analogous equation for the time $t_0 + T$, when the voltage on the capacitor plates is U_z:

$$U_z = \varepsilon\left(1 - e^{\frac{t_0+T}{RC}}\right). \tag{21.12}$$

From the last two equations we get:

$$\begin{aligned} t_0 &= RC\ln\varepsilon - RC\ln(\varepsilon - U_g), \\ t_0 + T &= RC\ln\varepsilon - RC\ln(\varepsilon - U_z). \end{aligned} \tag{21.13}$$

After subtracting the above equations by sides, we find the period T:

$$T = RC\ln\frac{\varepsilon - U_g}{\varepsilon - U_z}. \tag{21.14}$$

The expression $\ln[(\varepsilon - U_g)/(\varepsilon - U_z)]$ is a constant quantity for a certain voltage and a certain type of neon lamp. If we denote them with the symbol K, then equation (21.14) will take the form:

$$T = RCK. \tag{21.15}$$

We can see that the period of the relaxation vibrations is directly proportional to the capacity and resistance.

Capacitance measurement principle

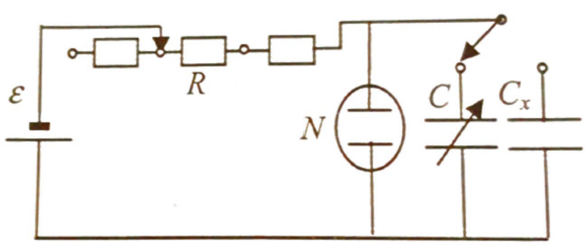


Figure 21.3. System for determining capacitance on the basis of relaxation vibrations

The formula (21.15) makes it possible to determine the capacitance if we can find the period of relaxation vibrations, the resistance of the circuit and the constant K. The measuring circuit is presented in Fig. 21.3.

The period is measured with a timer, observing the flashes of the neon lamp. We usually do not measure resistance - we use resistors marked with know value of R. In order to determine the constant K, instead of the tested capacitor, we take a series of capacitors with known capacities and measure the periods of relaxation vibrations. After measuring the period, we have all the quantities that determine the constant K and calculate it using the equation (21.15).

Measurements and Report:

Note: The capacitor measuring system has been additionally equipped with a digital oscilloscope connected to the computer. The oscilloscope allows you to measure the period of flashes (without using a stopwatch). The oscilloscope user manual is on the measuring stand. The lecturer decides whether flash periods will be measured using a stopwatch or oscilloscope.

1. At a certain resistance, change the reference capacity in steps. For each value, measure the time of 10-20 flashes and determine the period. Select the values of resistance and capacity so that the flashes are easy to count.
2. Repeat point 1 for other resistance values. The total number of combinations of resistance and capacity should be 20-30.
3. For each RC value, calculate the K constant, then its mean value and the standard deviation of the mean.
4. Take period measurements for unknown capacitors and calculate the capacitance of each.
5. Calculate the errors of each capacity, the easiest way is using the logarithmic differential method. Take the value calculated in point 3 as the error of the constant K.
6. Round off errors and results.
7. List the determined values and their errors as the final result.
8. Write down the final conclusions

Keywords:

- Electric capacity, capacitors
- Capacitor charging: Kirchhoff's law, change of current and voltage over time
- Capacitor discharge: Kirchhoff's law, change of current and voltage over time, time constant
- Neon lamp
- Relaxation vibrations: mechanism, graph, period
- Finding the constant K, determining the capacity of C_x

22. Investigation of the transformer

Exercise goals:

- Determining the transformer ratio
- Determining the dependence of voltage and transformer efficiency of current in the secondary winding of loaded transformer

Introduction

AC - Alternating current

The 50 Hz alternating current is commonly used to power domestic and industrial equipment. The dependence of alternating voltage of time t can be written with the following equation

$$u = U_0 \cos(\omega t), \tag{22.1}$$

where u is the temporary value of voltage, U_0 - peak voltage and ω - circular frequency. The cosine function argument is called a phase. In the closed circuit, the current with the same character of changes will flow, but its initial phase might be different from the voltage phase

$$i = I_0 \cos(\omega t), \tag{22.2}$$

where i is the temporary current value, I_0 - current peak and ϕ - voltage and current phase difference. The phase difference depends on the type and value of elements in the circuit: resistance, inductance and capacity of the circuit. If there is only resistance in the circuit, then $\phi = 0$, then there is no phase shift between voltage and current. If the circuit is inductive, the voltage is ahead of the current and the phase difference is within the range $< -\pi/2, 0)$. In the circuit which character is capacitive, the current is ahead of the voltage, and ϕ is in the range $(0, \pi/2 >$. The power generated in the alternating current circuit is expressed by the formula

$$P = \frac{1}{2} U_0 I_0 \cos(\phi), \tag{22.3}$$

The same power $P = UI$ would be created in the case of a direct current flow of voltage $U = U_0/\sqrt{2}$ and current $I = I_0/\sqrt{2}$ while $\phi = 0$. The U and I values are called effective voltage and effective intensity. By substituting effective values to equation (22.3), we get

$$P = UI \cos(\phi), \tag{22.4}$$

As can be seen in the case of alternating current, the power in the circuit does not only depend on the voltage and current, but also on their phase shift, which causes power losses. In electrical engineering, the expression $\cos\phi$ is called the cosine of the losses angle. In practice, we use effective voltage and current values. We read such values on meters. However, we must be aware that the voltage in the sockets of our homes varies in the range of 0 - 325 V (230 V is the effective voltage value).

Transformer

The transformer is a device commonly used in power engineering, electrical engineering, electronics, welding, etc. It serves to convert the voltage and the intensity of alternating current into other voltage and current without changing the frequency of the current. For example, transformers enable the exchange of high voltage used in power transmission lines (400 kV) for much lower voltage (230 V) used in home devices. Transformers, depending where they are used, have a diverse structure, and the theory connected with how these devices work is very complex. In this exercise, we will only learn basic, often simplified, informations related to this device. The transformer consists of a ferromagnetic core and at least two windings (coils) wounded on it (Fig. 22.1). Primary (supplying) and secondary (receiving) windings are electrical circuits of the transformer, while the transformer's core is a magnetic circuit. The way how transformer works is based on the phenomenon of electromagnetic induction. We distinguish three basic transformer operation states: *idle state*, *short-circuit condition* and *loaded state*.

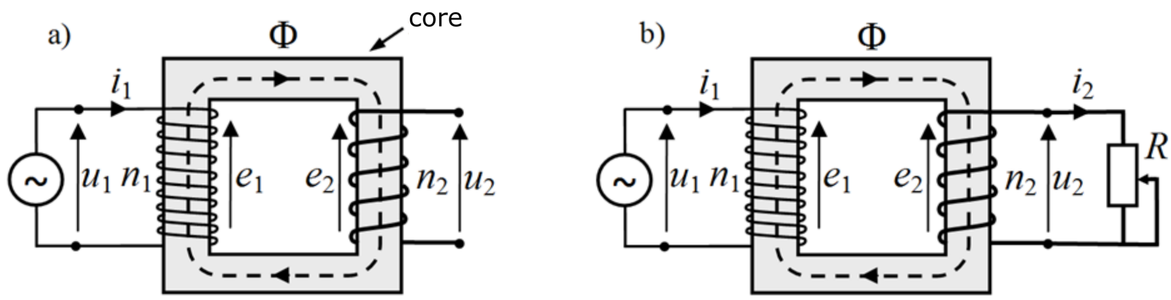


Figure 22.1. Overview of the transformer construction diagram a) idle state, b) loaded state

Idle state of the transformer

The idle state of the transformer is in a situation where the primary winding is connected to an alternating current source, and the secondary winding is open (Fig. 22.1). Alternating current flowing in the primary winding induces in the core an alternating magnetic flux Φ. According to Faraday's law of induction under the influence of changing magnetic field penetrating through the primary and secondary windings, temporary electromotive forces are induced in them of values

$$e_1 = -n_1 \frac{d\Phi}{dt}, \quad e_2 = -n_2 \frac{d\Phi}{dt} \tag{22.5}$$

where e_1 and e_2 are electromotive forces induced in primary and secondary windings, n_1 and n_2 are the number of windings in the primary and secondary windings, $d\Phi/dt$ is the derivative of a magnetic flux Φ after time t (the rate at which the flow changes in time). Usually resistances of the transformer windings are negligible, so in the idle condition of the transformer we can write that the temporary voltage drops on the primary and secondary windings are equal to the values induced in them by the electromotive forces

$$u_1 = e_1, \quad u_2 = e_2 \tag{22.6}$$

where u_1 is temporary voltage of the current source connected to the primary winding and u_2 is temporary voltage drop at the ends of the secondary winding. By using equations (22.5) and (22.6), the relation can be written

$$\frac{u_1}{u_2} = \frac{n_1}{n_2} \tag{22.7}$$

By replacing the instantaneous voltage drops on the primary and secondary windings with the corresponding effective voltages, we can finally write

$$\frac{U_1}{U_2} = \frac{n_1}{n_2} = K \tag{22.8}$$

The number K is called the transformer's transmission. Looking at equation (22.8) we can see that, if we choose the appropriate ratio of the number of primary and secondary windings, we can obtain an increase or decrease in voltage at the output of the transformer in relation to the supplying voltage.

Transformer short-circuit

The transformer's short-circuit condition (maximum load state) is in the case when the primary winding is connected to an alternating current source and the secondary winding is short-circuited. This corresponds to the situation in Fig. 22.1b, where the regulated resistor (receiver) is set to $R = 0$. Current i_1 flowing in the primary winding induces changing magnetic flux in the core. In the secondary winding under the influence of induction a temporary current i_2 appears. If we omit losses in the transformer, we can conclude, using the principle of energy conservation, that the power transmitted by the source to the primary winding $U_1 I_1$ is equal to the power transferred to the secondary circuit $U_2 I_2$.

$$U_1 I_1 = U_2 I_2 \tag{22.9}$$

where U_1, U_2 are effective voltages, and I_1, I_2 are effective currents in the primary and secondary windings, respectively. Using the above dependence and equation (22.8), we can write

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} = \frac{1}{K}\,. \tag{22.10}$$

Transformer load condition

So far, we have discussed two extreme cases of transformer operation status, when the secondary winding was open (receiver resistance $R = \infty$) and when it was shorted (receiver resistance $R = 0$). The transformer load state is referred to when an AC power source is connected to the primary winding and the secondary winding is connected to a receiver with resistance $R \neq 0$ (Fig. 22.1b). In this situation, the ratio of the voltages in the primary and secondary windings is not equal to the transformer ratio, because in the secondary winding circuit there is a voltage drop on the resistance of the secondary winding associated with the current flow. When we examine the voltage in the secondary winding, we observe that it decreases when we increase the value of current flowing in this winding (drop in the resistance of the receiver).

Transformer efficiency

The above considerations concern the so-called ideal (lossless) transformer. In a real transformer there are losses caused mainly by: winding resistance, eddy currents generated in the core, hysteresis of ferromagnetic and scattering of the magnetic field outside the core. Designers try to prevent these effects by: using high-quality copper wires in the windings, creating transformer cores from a series of isolated ferromagnetic sheets, optimizing the shape of the core and windings depending on the transformer use. An important parameter of the transformer is its efficiency calculated as the ratio of the power P_2 to the power taken from the source P_1. To show the η efficiency in percent, we use the equation

$$\eta = \frac{P_2}{P_1} \cdot 100\% \,. \tag{22.11}$$

Remembering that we are dealing with alternating current, and the circuits have an inductive character, the formulas for powers P1 and devoted P2 can be written in accordance with equation (22.4) as

$$P_1 = U_1 I_1 \cos\phi_1 \,,\; P_2 = U_2 I_2 \cos\phi_2 \,, \tag{22.12}$$

where ϕ_1 and ϕ_2 are the angles of the phase shift between voltage and current in the primary and secondary circuits. Assuming with an extremely big approximation that $\phi_1 = \phi_2$, we can write an approximate formula for the efficiency of the transformer

$$\eta = \frac{U_2 I_2}{U_1 I_1} \cdot 100\% \tag{22.13}$$

In order to obtain the maximum efficiency of the transformer, it should be properly designed taking into account the electrical parameters of the receiver. Well-designed transformers of very high power achieve efficiency of 97-99%, while in the case of simple transformers of low power an efficiency is around 80%.

Measuring system

A measuring system consisting of an adjustable AC power supply, transformer, slider resistor, switch, two voltmeters and two ammeters (Fig. 22.2) was designed for transformer testing. The transformer windings have taps

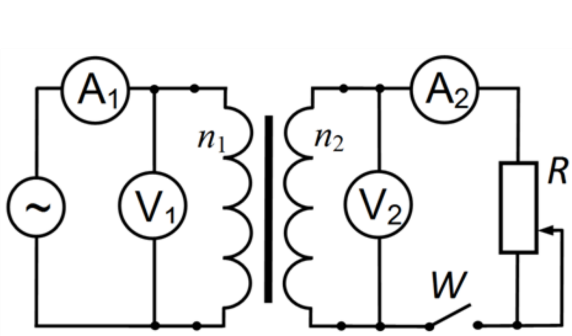


Figure 22.2. Scheme of the electric measuring diagram

enabling selection of 200, 400 or 600 turns. The voltage of the power source can be changed in the range of 1-12 V. The resistor acting as a receiver enables resistance changes in the range of 0-34 Ω.

Attention! There is a possibility of strong current pulses in the circuit during connecting and switching off the transformer windings. Therefore, the change in the number of transformer windings and the switching on and off of the power supply of the circuit should be carried out with a minimum voltage at the power source (1 V).

Measurements:

A. Transformer tests in the idle state - determination of the transformer ratio

1. Connect the circuit according to the diagram in figure 22.2, selecting the number of turns $n_1 = 400$ and $n_2 = 600$. Switch W turn to position - disconnected (0).
2. Set the power supply voltage adjustment knob to 1 V and turn it on.
3. Turn on the multimeters and additionally press the blue buttons on them to select the measurement of alternating currents and voltages (AC mode).
4. Change the supply voltage every 1 V in the range 1 - 10 V, each time saving voltage U_1 and U_2.
5. Repeat the measurements for the number of secondary turns $n_2 = 400$ and $n_2 = 200$ (when $n_1 = 400$).
6. Plot the coordinates of dependences of secondary voltage and primary voltage $U_2 = f(U_1)$ on one coordinate system
7. Using the obtained measurement results and equation (22.8), determine the tested transformer transformations and then their average values and measurement uncertainties.
8. Determine the theoretical values of the transformer ratio from the ratios of the number of windings on the primary and secondary windings (equation 22.8). Compare experimental and theoretical results.

B. Transformer tests in the short-circuit condition

1. Set the power supply voltage to 1 V, turn the W switch to the position - switched on (1) and the resistor slider to the 0 Ω position.
2. Take measurements of the currents I_1 and I_2 like in the previous tests ($n_1 = 400$ and $n_2 = 600$, 400, 200). Each time after the measurements, the voltage on the power supply should be set to 1 V.
3. On one coordinate system, plot the dependence of the secondary current intensity of the primary current $I_2 = f(I_1)$.

C. Transformer tests in the loaded state

1. set $n_1 = 400$, $n_2 = 200$;
2. set power voltage to 4 V and the "W" switch to "1";

3. Perform 12 - 15 measurements of U_1, I_1, U_2 and I_2 values, changing the receiver's resistance in the range of 0 - 34 Ω. The resistance in the beggining should be changed every 1 Ω, then every 2 Ω, and in the end every 4 Ω.
4. Switch W switch to position - disconnected (0), then write values U_1, I_1, U_2 and I_2 (in this case $R = \infty$).
5. Set the PSU controller in the 1 V position, and then turn off the PSU.
6. Plot the dependence of voltage of the current in the secondary circuit $U_2 = f(I_2)$.
7. Using the measurement results and equation (22.13), calculate the transformer efficiency for individual measurements, then plot the dependence of the transformer efficiency of the current intensity in the secondary winding $\eta = f(I_2)$.
8. Write down the final conclusions

Keywords:

- working principle of an electrical transformer,
- alternating/direct current (AC/DC),
- power rating, ratio and efficiency of a transformer,
- load/no-load state, open/short circuit.

23. Determining the dependence of conductivity on temperature for semiconductors and conductors

Introduction

According to Ohm's law in its most general form, the current density anywhere in a conductive material is directly proportional to the electric field strength.

$$j = \sigma E. \tag{23.1}$$

In the above equation, j is the current density (the ratio of the current to the cross-sectional area), E - the electric field strength. The proportionality factor o is called electrical conductivity. The conductivity value is directly determined by the concentration and mobility of the charge carriers.

$$\sigma = e(n\mu_e + p\mu_p). \tag{23.2}$$

The concentration of n electrons and holes p is defined as the number of these carriers per unit volume, and the mobility (electrons - μ_e, holes - μ_p) is the ratio of the velocity of lifting to the electric field strength.

In semiconductors, both concentration and mobility depend on the type of material and temperature, so conductivity also depends on these parameters. In conductors (metals) the concentration of carriers (only electrons are important) is very high and does not depend on temperature, and the temperature dependence of conductivity is determined by the decrease in mobility with increasing temperature. This relationship is usually expressed by resistance ($R \approx 1/\sigma$) and for metals it has the form:

$$R = R_0[1 + \alpha(T - T_0)], \tag{23.3}$$

where R_0 is the resistance at T_0 and α is the mean temperature coefficient of resistance. Formula (23.3) should only be used in a not too large range of temperature values, since the coefficient α changes with temperature.

In semiconductors, the current carriers are electrons in the conduction band and holes in the valence band. The electrons are delivered to the conduction band either from the valence band (in intrinsic semiconductors) or from doping-donor levels (in n-type semiconductors). On the other hand, holes are formed in the valence band after the electron passes either to the conduction band or to the doping-acceptor levels (in p-type semiconductors).

The electronic transitions in semiconductors are shown in Fig. 23.1. The number of electrons going to a higher energy level depends exponentially on the difference in levels and on temperature and is expressed in the case of intrinsic semiconductors:

$$n = n_{0s} e^{\frac{-E_g}{2k_B T}}, \tag{23.4}$$

where: E_g - bandwidth (see Fig. 23.1), k_B - Boltzmann constant, T - absolute temperature.

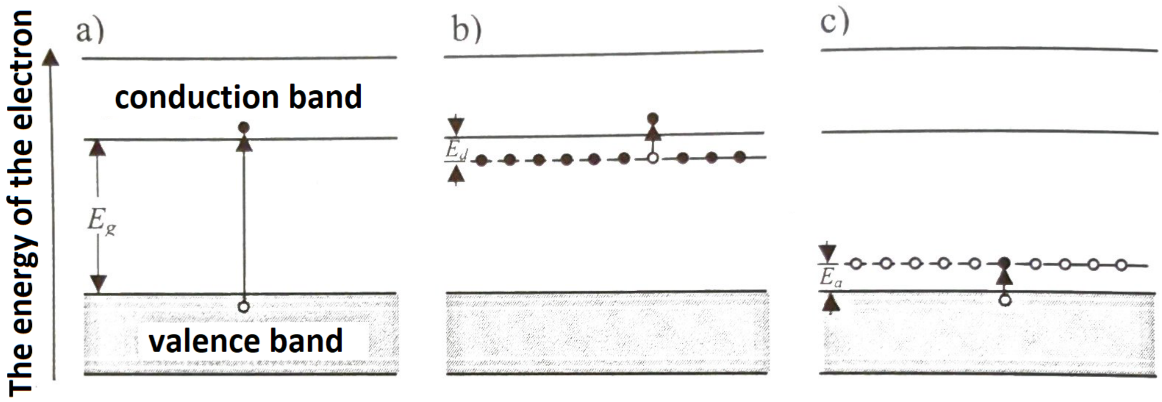


Figure 23.1. Bands and energy levels in intrinsic semiconductors (a), n semiconductors (b) and p type semiconductors (c); full circles - electrons, empty circles - holes, E_a, E_d - energy of acceptors and donors

Due to the fact that each electron in the conduction band has a free hole in the valence band, the concentrations of both types of carriers are the same: $n = p$. In the case of doped semiconductors, the carrier concentrations are determined by the energy differences E_d and E_a and by the temperature:

$$n = n_{0d} e^{\frac{-E_d}{2k_B T}}, \qquad n = n_{0a} e^{\frac{-E_a}{2k_B T}} \tag{23.5}$$

As the temperature rises, the number of carriers coming from the bellows levels also increases until all electrons leave the donor levels or fill the acceptor levels. Further increasing the temperature does not lead to an increase in the concentration of carriers. In this temperature range, the number of intrinsic carriers is still very small - so we observe the phenomenon of *impurity saturation.* Only at higher temperatures, carriers from interband transitions begin to dominate and concentration begins to increase rapidly.

The mobility of carriers, as in metals, decreases with temperature. However, these changes are much slower than changes in concentration, so we can assume that the conductivity depends on the temperature as well as the concentration of the carriers.

After taking into account equations (23.4) and (23.5) in formula (23.2), we can express the temperature dependence of the conductivity in the form:

$$\sigma = C_1 e^{\frac{-E_g}{2k_B T}} + C_2 e^{\frac{-E_{dom}}{2k_B T}}, \tag{23.6}$$

where by E_{dom} we mean one of the quantities E_d or E_a, depending on the type of semiconductor. Constants C contain mobility and the size of n_o. At a sufficiently low temperature, the first component in formula (23.6) may be neglected, while at high temperature, when the impurity levels become saturated, the second component may be neglected. In the first case

$$\sigma_{dom} = C_2 e^{\frac{-E_{dom}}{2k_B T}}, \tag{23.7}$$

in the second

$$\sigma_{sam} = C_1 e^{\frac{-E_g}{2k_B T}}. \tag{23.8}$$

Both of the above equations have the same mathematical structure, so it's convenient to replace them with one common equation

$$\sigma = C e^{\frac{-E_A}{k_B T}}. \tag{23.9}$$

where $E_A = E_g/2$ for intrinsic conductivity or $E_A = E_{dom}/2$ - for dopant conductivity. The magnitude of E_A is called *activation energy.*

The temperature dependence of the semiconductor's conductivity is most conveniently analyzed by plotting this dependence on a semi-logarithmic scale. After logarithm the formula (23.9) we get the expression:

$$\ln \sigma = \ln C - \frac{E_A}{k_B} \cdot \frac{1}{T}. \tag{23.10}$$

If we now put the reciprocal of the temperature on the abscissa and $\ln \sigma$ on the ordinate axis, then the graph of the temperature dependence of the semiconductor will be a straight line with the slope coefficient E_A/k_B. In a wide range of temperature values, including intrinsic and intrinsic conductivity, the graph will appear as a broken line (Fig. 23.2). The sections with different slopes correspond to different activation energies.

Measurements and calculations

Old description - introduction to measure the resistance R by compensation method

In order to determine the searched relationships, we measure the electrical resistance of a wire and a semiconductor (thermistor) at different temperatures. The tested little ones are placed in the ultrathermostat and their resistances are measured with the Wheatstone bridge. The structure of the Wheatstone bridge is shown in fig. 23.4. The main operation when using the bridge is to select the resistance R (it consists of a series of resistors in a decade system) in such a way as to obtain an equilibrium of the bridge consisting in resetting the current flowing through the galvanometer G. The condition for equilibrium is the equality of the electric potentials in the points B and D. When counting the potential in relation to the point C, we will express the above condition as:

$$Ri_x = R_2 i. \tag{23.11}$$

A similar condition exists for the ABD branch:

$$R_x i_x = R_1 i. \tag{23.12}$$

We divide both equations side by side and find the resistance we are looking for:

$$R_x = \frac{R_1}{R_2} R. \tag{23.13}$$

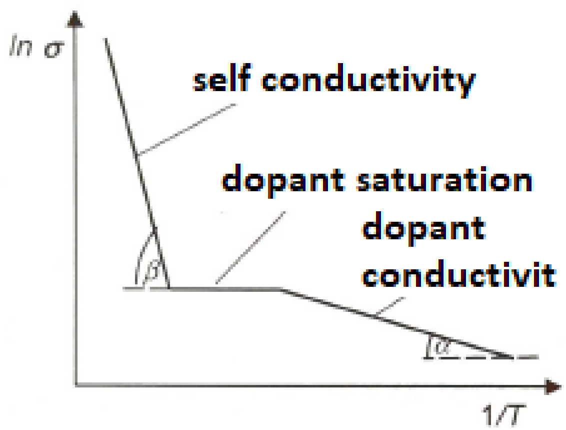


Figure 23.2. Logarythm of conductivity as a function of the reciprocal of temperature

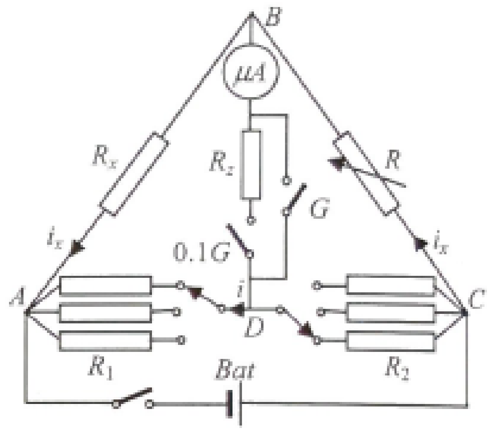


Figure 23.3. Structure of a laboratory Wheatstone bridge

The values of the resistors R_1 and R_2 are a sequence of powers of 10, e.g. 1, 10, 100, 1000. These resistors allow the resistance to be measured over a very wide range.

In order to find the appropriate settings for R_1 and R_2, first set the maximum values of R_1 and R_2. Then we change the Ri values from the minimum to the occurrence of a change in the direction of the galvanometer deflection. If the deflection direction does not change in the whole range, set the lowest value of R_2 and change R_1 again from the minimum until the value at which the deflection direction changes.

After finding the settings of R_1 and R_2, we leave them unchanged until the end of the measurement of a given resistor, and further adjustment is carried out using the resistance R. Using the decade resistance knobs, starting with the largest, we narrow the range in which the galvanometer deflection changes direction, until the galvanometer deflection is zero.

The $0.1G$ button turns on the galvanometer by the safety resistance R, which reduces the sensitivity of the galvanometer. In order to balance the bridge more precisely, we press the G button and repeat the activities related to reaching zero excursion, without changing the greatest decade. After achieving the equilibrium, we turn off the power source and check the zero indication of the galvanometer.

The design and operation of the ultrathermostat are described in chapter 7. Instead of the ultrathermostat, the resistors can be placed in a water bath and heated electrically.

Present description - introduction to measure the resistance R by multimeter H 2105/B

The measurements of resistance of conductor and semi-conductor is done directly by the multimeter H2105/B (see Figure 23.4). In figure 23.4 are presented two multimeters: on the left for measure the semiconductor resistance, on the right side the conductor resistance. Both multimeters are working in "eco" mode which is use to safe energy consumption. Every 15-20 minutes the multimeters are switch off. In such case you need to press the red button on the left side below the digital displayer. In semiconductor case the range is set to 2 MΩ and show the value of $R = 0.227$ MΩ $=$ 227 kΩ. When the temperature of semiconductor reach the 25-28 °C, you can change the range on multimeter for 200 kΩ and continue the measurements up to temperature 90°C. In the conductor case the multimeter operate on the range of 200 Ω in the full range of temperature and in Fugure 23.4 present the value of $R = 108.5$ Ω. To estimate the uncertainty of R reading the Table 5.1 present the typical accuracy values of multimeter.

Table 5.1. Accuracy of resistance measurement with the H2105/B meter

Range	Resolution	Precision
200 Ω	0.1 Ω	±(0.8% of reading + 0.3) Ω
2 kΩ	1 Ω	±(0.8% of reading + 2.0) Ω
20 kΩ	10 Ω	±(0.8% of reading + 20) Ω
200 kΩ	100 Ω	±(0.8% of reading + 200) Ω
2 MΩ	1 kΩ	±(0.8% of reading + 2.0) kΩ
20 MΩ	10 kΩ	±(1.0% of reading + 50) kΩ

For the measured values of R presented in Fugure 23.4 the accuracy of resistance can be expresed as:
semiconductor $R = 0.2270 \pm 0.0038$ MΩ or $R = 227.0 \pm 3.8$ kΩ,
conductor $R = 108.5 \pm 1.7$ Ω.

Figure 23.4. multimeter H 2105/B

Equation (23.10) can be used to find the E_A activation energy. If we apply the relation $\sigma \propto 1/R$, we get the equation:

$$\ln \frac{1}{R} = \ln C - \frac{E_A}{k_B} \cdot \frac{1}{T}, \tag{23.14}$$

which in the graph $\ln(1/R) = f(1/T)$ represents a straight line with a slope coefficient

$$a_{reg} = \frac{E_A}{k_B}, \tag{23.15}$$

We can find the same slope factor from linear regression. When it is known, then the last equation makes it possible to calculate E_A.

Measurements:

1. Measure resistance R for both conductor and semiconductor in the temperature range from the room temperature up to 90°C with temperature change step 5°C. Use the digital multimeters for the measurements of resistance. Please set the correct range of multimeters for conductor and semiconductor.

Report:

1. Plot $R = f(Temp.)$ for both conductor and semiconductor at the same graph for comparison; feel free to use different scales if needed;
2. For a semiconductor: calculate $ln(1/R)$ and $1/Temp.$ and plot those functions; temperature need to be expressed in Kelvins;
3. Using linear regression determine the slope coefficient a_{reg} and its error Δa_{reg};

4. Try to determine the energy doping level according to the formula: $a_{reg} = E_A/k_B$, where: E_A – activation energy, k_B – Boltzmann constant. The doping level can be calculated approximately; the function $ln(1/R) = f(1/T)$ (see above) is linear and we can use the linear regression to determine the slope coefficient a_{reg}. Regression can be done by using e.g. gnuplot or an office suite such as Libre Office. Having a_{reg}, we can finally calculate E_A from the relation $a_{reg} = E_A/k_B$; (see Table B.1 for k_B value)
5. Try to calculate complex error of E_A given above (exact differential);
6. Round the calculated values and present the final form of the result E_A [in units of J and eV].
7. Write down the final conclusions

Keywords:

- Ohm's law, conductivity, concentration, mobility, factor temperature resistance
- Free carriers in an intrinsic and doped semiconductor
- The dependence of conductivity on temperature
- Wheatstone Bridge
- Calculation of the position of the admixture level or the width of the forbidden gap, electron-volt (eV)

24. Investigation of the influence of the magnetic field on a conductor with current

Exercise goals:

- Getting to know the phenomenon of influence of magnetic field on the conductor with the current
- Determining the dependence of electrodynamic force on the current flowing in the wire frame and of the amount of frame windings
- Determination of an average value of an induction of the magnetic field between the poles of a magnet

Introduction

The impact of a magnetic field on a conductor with the current is a common phenomenon used in technology. An example of such an application are electric motors that, among the others, are used in trams, washing machines, hair dryers, car wipers, toys. Ability to do work by an electric motor comes from the emergence of a force (the so-called electrodynamic force) acting on the conductor with the current in the magnetic field. Before we start discussing about an electrodynamic force, let's start with a force and how is it acting on a charged molecule moving in a magnetic field.

Lorentz force

In many experiments with molecules having an electric charge moving in a magnetic field, a force that caused bending of their track was observed. Dutch physicist Hendrik Lorentz was the first to write down the following formula describing this force (the so-called Lorentz force)

$$\vec{F}_L = q(\vec{v} \times \vec{B}), \tag{24.1}$$

in which: q - particle charge, $\vec{v}$ particle velocity vector, $\vec{B}$ magnetic field induction vector. By writing down the same formula in scalar form, we get the value of the force acting on the molecule

$$F_L = qvB \cdot \sin\alpha, \tag{24.2}$$

in which: α is angle between $\vec{v}$ and $\vec{B}$ vectors. By analyzing formulas (24.1) and (24.2), it is easy to see that the F_L force equals zero when the charged molecule is not moving or moves along the direction of the magnetic field lines, while the maximum is when the molecule moves perpendicular to the direction of the magnetic field lines. The direction of Lorenz's force is always perpendicular to the plane created by $\vec{v}$ and $\vec{B}$ vectors and the orientation depends on the charge sign (Fig. 24.1a).

Electrodynamic force

The consequence of the Lorentz force is the force acting in the magnetic field on the conductor with the current. Fig. 24.1b shows a part of the conductor in which the current flow is caused by the free electrons movement. In the section of length of l, at a given moment, n electrons with charge e and average velocity of lifting v_u the Lorentz force acts on each electron (Fig. 24.1a, b) with a value

$$F_L = ev_uB \cdot \sin\alpha. \tag{24.3}$$

The total force acting on the charges in a segment of conductor of the length l equals

$$F_{ED} = nev_uB \cdot \sin\alpha. \tag{24.4}$$

The sum of free charges in the conductor section is $q = ne$, and the flow time of the electron through the segment of length l is $t = l/v_u$, so using the relation on the current $I = q/t$, we get the equation

$$I = \frac{nev_u}{l}. \tag{24.5}$$

Based on the formulas (24.4) and (24.5), the relation can be written

$$F_{ED} = IlB \cdot \sin\alpha, \tag{24.6}$$

or in vector form

$$\vec{F}_{ED} = I(\vec{l} \times \vec{B}). \tag{24.7}$$

Formula (24.6) describes the value of electrodynamic force. The vector $\vec{l}$ has a direction and an orientation in the direction of the current flow in the conductor (this flow is opposite to the direction of electron motion). The angle α in formula (24.6) is the angle between the vector of the conductor section $\vec{l}$ and the magnetic field induction vector $\vec{B}$ (Figure 24.1c).

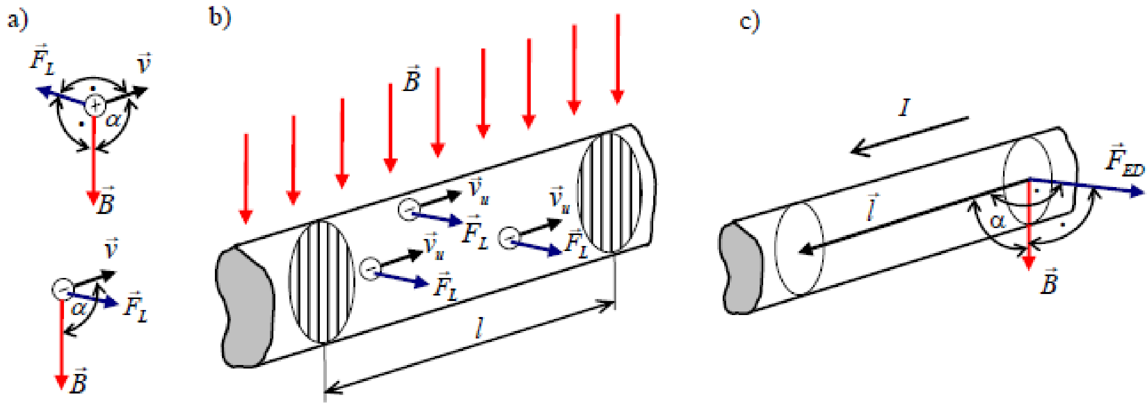


Figure 24.1. a) shared orientation of vectors: $\vec{v}$ - velocity of charged molecule, $\vec{B}$ - magnetic induction and $\vec{F}_L$ - Lorentz force, b) direction of force acting on individual electrons in a section of conductor with current placed in a magnetic field, c) direction and orientation of electrodynamic force $\vec{F}_{ED}$ acting on section of the conductor with current of intensity I (arrow under I means direction and orientation of current)

Description of the experiment setup

On the measuring stand (Fig. 24.2) there is a horseshoe magnet that generates a very strong magnetic field (Note: Do not bring metal objects or electronics close!). Above it, on a rotary axis, a rectangular wire frame was hung so that its bottom side was between the poles of the magnet. The frame, which was created from coiling 25 windings of a copper wire, has choice of 5, 15 and 25 windings. This allows to connect a power source to 5, 10, 15, 20 or 25 frame windings depending on how the wires are connected. The source of current in the circuit is the regulated DC power supply. The electrical circuit also includes: an ammeter that allows precise measurement of the current and a resistor that is protecting the frame from overheating. A miniature laser was attached to the frame, the ray of which serves as an indicator enabling reading on the scale under the frame.

Method of determining electrodynamic force

In order to determine the electrodynamic force acting on the bottom side of the frame, we need to consider moments of forces occurring after the frame is leaned out from the equilibrium position. These moments will be related to the forces of: gravity and electrodynamics. Figure 24.3a shows schematically a rectangular frame with sides

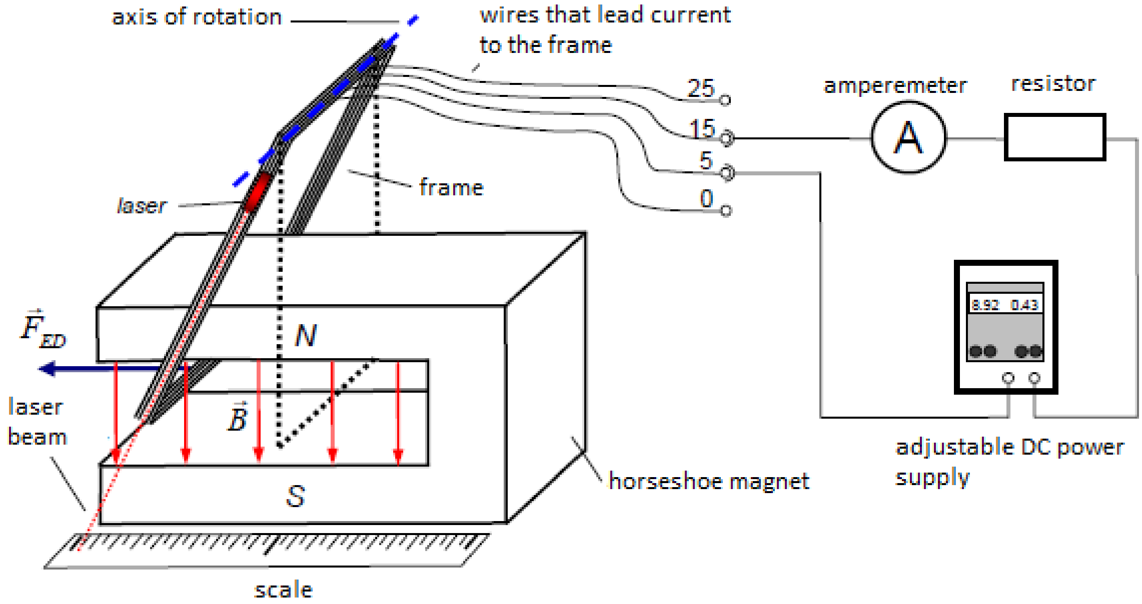


Figure 24.2. Scheme of the exercise setup. In the above example, the current flows through 10 frame windings (15 - 5 = 10)

of length a and b deflected from the equilibrium position by angle Φ. Middle of mass of three sides are marked on them m_b and m_a. Figure 24.3b shows the gravitational force acting on the center of mass of individual sides and the electrodynamic force. In addition, the arms vectors r associated with particular forces are marked.

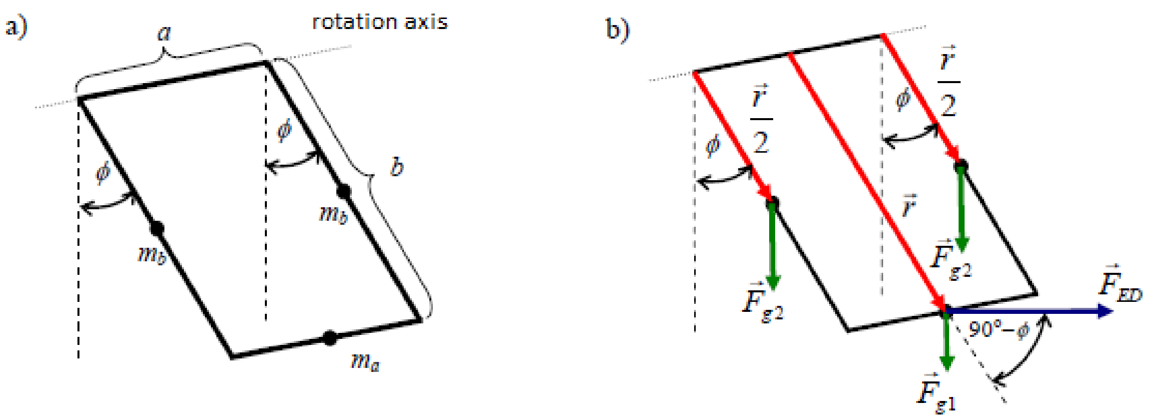


Figure 24.3. The wire frame a) frame scheme with mass centers of individual sides marked, b) strength and force arm vectors

For the frame to stay in balance, the moments of forces acting on it should balance out each other

$$\vec{r} \times \vec{F}_{g1} + \frac{1}{2}\vec{r} \times \vec{F}_{g2} + \frac{1}{2}\vec{r} \times \vec{F}_{g2} + \vec{r} \times \vec{F}_{ED} = 0. \quad (24.8)$$

The values of gravitational forces can be calculated by multiplying the mass of individual sides of the frame m_a, m_b by gravitational acceleration g ($F_g = mg$), while the arm vector r is equal to the side length b. Using the above equations and trigonometric relations we can write the scalar form

$$(b \sin\phi) m_a g + 2(\frac{b}{2} \sin\phi) m_b g - (b \sin(90^\circ - \phi)) F_{ED} = 0. \quad (24.9)$$

After transformations, we obtain the following formula for electrodynamic force

$$F_{ED} = (m_a + m_b) g \cdot \tan\phi. \quad (24.10)$$

Because the mass m of the entire frame is equal to the sum of masses of its individual sides $m = 2m_a, +2m_b$, we can ultimately write

$$F_{ED} = \frac{1}{2} mg \cdot \tan\phi. \quad (24.11)$$

Given the mass of the frame, the acceleration of the earth and the tangent of the angle of deflection of the frame makes it possible to determine the electrodynamic force. In the described exercise, we will use a beam of light emitted by a laser and a scale with a millimeter scale to determine $\tan\phi$ (Figures 24.2 and 24.4).

In Figure 24.4 it can be seen that $\tan\phi$ is equal to the ratio of the length of segment x read from the scale to the distance of the axis of rotation from the center of the scale d. Thus, the value of the electrodynamic force is equal to

$$F_{ED} = \frac{1}{2} mg \frac{x}{d}. \quad (24.12)$$

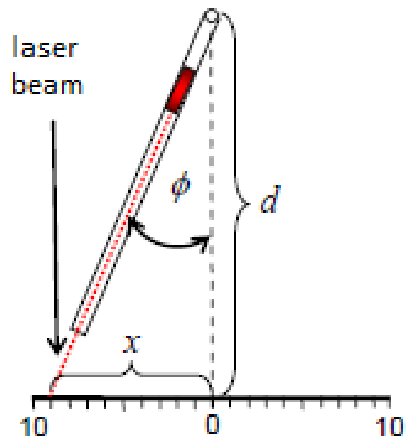


Figure 24.4. The Scheme helpful to determination of $\tan\phi$

It is easy to notice that the quantities m, g and d are constant, so by entering the factor $c = mg/2d$, we can ultimately write that

$$F_{ED} = cx. \quad (24.13)$$

It is possible to see that the electrodynamic force is directly proportional to the indications on the x scale. Experimental proportionality factor $c = (2.65 \pm 0,05)$ N/m. In the above considerations, it was assumed that the moment of gravitational force acting on a miniature laser is negligibly small.

Determination of induction of a magnetic field value between a magnet poles

In the described experiment, it can be seen that the current flow direction in the lower side of the frame is perpendicular to the direction of the magnetic field lines ($\alpha = 90°$). The formula (24.6) can therefore be written in the form

$$F_{ED} = IlB. \tag{24.14}$$

where l is the total length of the conductor with the current interacting with the magnetic field. This length can be determined by multiplying a number of windings and a length of the lower side of a frame, $l = na$, where $a = (13,0 \pm 0,2)$ cm. To determine the average value of induction of a magnetic field interacting with a lower side of the frame, we can use equation

$$F_{ED} = BnaI. \tag{24.15}$$

By substituting $y = F_{ED}$, $x = I$ and $a_{reg} = Bna$ in the above equation, a linear relationship of type $y = a_{reg}x + b$ is obtained. It is a linear relation where the value of a_{reg} is directional ratio of straight line. By plotting the dependency of an electrodynamic force as a function of a current flowing through the frame: $F_{ED} = f(I)$,a straight line should be obtained. Applying the linear regression method to a graph obtained this way, one can determine the slope modulus of a a_{reg} line and then, using known values of n and a, determine value of magnetic field induction B. The calculated value of induction of the magnetic field is actually a certain average value, because the magnetic field in which moves the bottom side of the frame is not completely homogeneous over the entire length of the side.

The course of the exercise

A. Determination of dependence of electrodynamic force on the intensity of current flowing through frame winding $F_{ED} = f(I)$.

1. Connect the circuit according to the diagram shown in Fig. 24.2, selecting 5 frame windings (wires connected to inputs 0 and 5). Make sure that the multimeter knob is set to the ammeter position, then turn on the power supply and the laser.
2. By adjusting the power supply, increase the current flowing through the frame winding so that the position of the laser spot changes every 1 cm. Each time, write the position of the spot on the scale (1 - 10 cm) and the corresponding intensity of current values. Set the power supply voltage to zero, change the wires in places and repeat the measurements for the opposite direction of the current flow.
3. Connect wires in different combinations, repeat measurements from point 2 for the number of windings 10, 15, 20 and 25.
4. Using the equation (24.13), calculate the F_{ED} electrodynamic force corresponding to the individual frame deflections.
5. On the common graph, plot the dependence of the electrodynamic force on the intensity of the current flowing through the windings of the frame $F_{ED} = f(I)$.

6. Note the conclusions. Warning! After each completed measuring series, set the voltage of the power supply to zero!

B. Determine the electrodynamic force as a function of coil turns, $F_{ED} = f(n)$

1. Using the obtained $F_{ED} = f(I)$ graphs, read approximate F_{ED} values for one constant current value in the range of 0.15 - 0.25 A (for example 0.2 A).
2. For $I = const$ plot the dependence of electrodynamic force on the number of frame windings $F_{ED} = f(n)$.
3. Note the conclusions.

C. Determine the magnetic field induction B between the magnetic poles

1. Select the $F_{ED} = f(I)$ diagram closest to the straight line and then, using the linear regression method, determine its slope a_{reg} factor and error.
2. Using the data and the formula (24.15), determine the value of magnetic field induction ($B = a_{reg}/na$).
3. Check the units for all physical quantities and calculate the measurement error of the calculated induction of the magnetic field (logarithmic or absolute differential method).
4. Write down the final conclusions

Keywords:

- magnetic field,
- Lorentz force,
- the electrodynamic force.

25. Determination of the Planck constant and output work based on photoelectric effect

Introduction

Generally, no electric current flows in an electrical circuit comprising a voltage source and two metal plates separated by a vacuum layer. However, if a plate with a negative potential is illuminated, a current will appear, called a *photocurrent* - the greater, the stronger the lighting will be. This phenomenon is called *photoelectric.* His research showed that:

- microscopically the phenomenon consists in knocking out electrons from the metal surface by the incident light, - photocurrent appears immediately after the exposure of the metal (after $\approx 10^{-9}$s);
- photocurrent intensity, i.e. the number of electrons emitted per unit time, it is proportional to the illuminance;
- photoelectron energy does not depend on the illuminance; it is proportional to the oscillation frequency of the light wave;

- Photocurrent appears only when the radiation frequency exceeds a certain limit.

The above properties can only be explained at the microscopic level on the basis of the quantum theory of light. In solids, which are conductors, the valence electrons are not bound to the parent atoms - they move freely in the crystal lattice, creating the so-called electron gas. The free movement of electrons in metallic crystals results from the *potential energy distribution.* As a result of the interaction of atoms (marked with circles in Fig. 25.1 with a plus), the potential barriers separating adjacent atoms are reduced to a value lower than the total energy of the electron and do not prevent the electrons from moving (black ball with a minus).

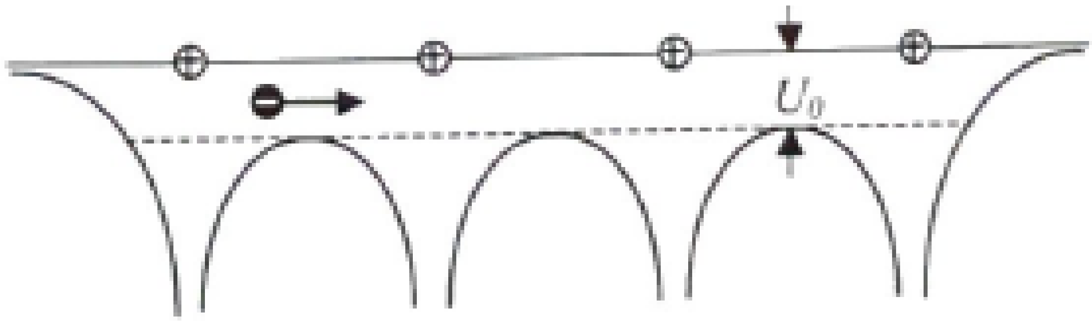


Figure 25.1. Potential energy in the crystal

The atoms on the crystal surface (extreme left and right) have neighbors only on the inside and therefore the potential energy in the vicinity of these atoms is different than in the back of the crystal. The potential energy at the surface is greater, so the surface is a barrier to the electrons and they cannot leave the crystal. Graphically, it can be said that the electrons are trapped in a potential "box" - they can move freely inside it, but cannot pass through its walls.

It is possible for the electron to leave the metal (breaking the potential U_0 barrier) if it obtains additional energy of at least eU_0. This energy is called the *exit work.*

The source of energy can be:

- increased temperature,
- then the phenomenon of thermo-emission occurs,
- strong electric field - field emission takes place,
- particle bombardment with sufficiently high kinetic energy and
- crystal lighting.

In the latter case, we are dealing with a photoelectric phenomenon. The ejection of an electron from a metal by a photon occurs only when the energy of the photon hv is equal to or greater than the work of the output W. The frequency corresponding to this condition is the above-mentioned limiting frequency The energy transformation in the photoelectric effect is described by the Einstein equation:

$$h\nu = W + \frac{1}{2}mv^2, \tag{25.1}$$

where: h - Planck constant equal to $6.62 \cdot 10^{-34}$ J·s, ν - frequency of the light wave, W - output work, m - electron mass, v- its speed outside the metal. This equation should be treated as an energy balance - the energy of the incident photon is converted into the work of the output and the kinetic energy of the electron.

The photoelectric effect has found practical application in photocells. The construction of the photocell is shown in fig. 33.2. It consists of a glass bulb, the back wall of which is covered inside with a metal layer with a low exit function. In the center of the bulb is a wire loop that acts as the anode. Depending on the content, the photocell banks can be vacuum or gas. In a vacuum photocell, the total current consists of electrons

Figure 25.2. Construction of the photocell; A - anode, K - cathode

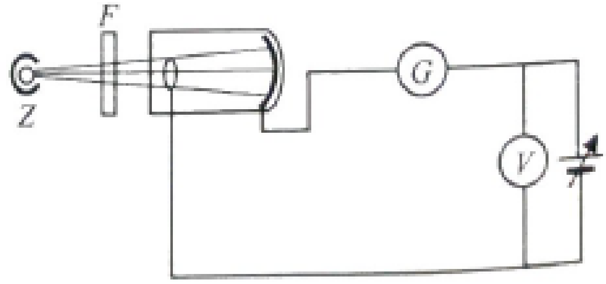


Figure 25.3. System for testing the characteristics of the photocell; Z - light source, F - filter, G - galvanometer.

knocked out of the cathode and attracted to the anode. The amperage is relatively low.

Higher current intensity is obtained in gasified photocells, filled with a small amount of noble gas, in which primary photoelectrons can ionize the gas atoms, thus increasing the number of current carriers.

The electrons knocked out of the cathode of the vacuum photocell create an electron cloud that repels the next electrons moving towards it. As the voltage at the anode increases, the cloud is drawn more and more to the anode until each photoelectron reaches the anode at a certain voltage. Despite the further increase in voltage, the intensity of photocurrent does not increase further - the saturation state has been reached. To get more photocurrent, you need to increase the illumination.

The photoelectric current flows even when there is no voltage between the anode and cathode. This is due to the kinetic energy possessed by the electrons when they are knocked out of the metal. A complete loss of current can be caused by applying a voltage of opposite polarity, i.e. a lower potential, to the anode. If the voltage is of the appropriate value, called the inhibitory potential V_h, the electrons are completely inhibited - their kinetic energy is used to perform work against the electric field:

$$\frac{1}{2}mv^2 = eV_h. \tag{25.2}$$

Taking into account the above relationship, we can transform equation (25.1) into the form

$$V_h = \frac{h}{e}\nu - \frac{W}{e}. \tag{25.3}$$

It can be seen from the above that the voltage needed to inhibit photoelectrons is the greater, the higher the frequency of the illuminating radiation.

Measurements and calculations

In this exercise, we perform two tasks:

- Determine the current-voltage characteristics of the photocell with the system shown in fig. 25.3.
- We determine Planck's constant from the braking voltage measurements for lighting of different wavelengths.

The photocell is illuminated by the light of the Z filament lamp which passes through a suitable filter F which transmits only light of a given wavelength. We have various filters at our disposal, so we can choose different wavelengths. The regulated voltage is supplied to the photocell from the DC power supply. The current is measured with a microammeter or a G galvanometer. Instead of a galvanometer, you can use a suitable resistor and a voltmeter connected in parallel to it, which measures the voltage caused by the flow of photocurrent. We calculate the value of photocurrent from Ohm's law.

In order to determine the braking voltage V_h, set a small positive voltage (at the anode) and slowly reducing it, bring it to the zero value of photocurrent. This will happen at some negative voltage, which is the brake voltage. Before starting these measurements, carefully determine the meter zero deflection by reading the current indication with the meter input disconnected and shorted. We take the absolute value of the braking voltage for the graph and for calculations.

Proceeding in this way, for different wavelengths we obtain data for the plot $V_h = f(\nu)$. From the $V_h = f(\nu)$ plot we can find the Planck constant h and the work of the output W. The slope of the line, described by the equation (25.3), is h/e, and the point of intersection with the ordinate axis is W/e. The same parameters of the line - let's denote them a_{reg} and b_{reg} respectively - can be calculated by applying linear regression to measurement points. From the comparison we get:

$$\frac{h}{e} = a_{reg}, \quad -\frac{W}{e} = b_{reg}. \tag{25.4}$$

After transforming the above equations, we get the final form of the expressions into Planck's constant and the work of the output

$$h = a_{reg}e, \quad W = b_{reg}e. \tag{25.5}$$

Measurements:

1. The measuring system should be connected in accordance with the diagram attached to the exercise (see diagram).

Figure 25.4. Multimeter V562 for measurement U_1.

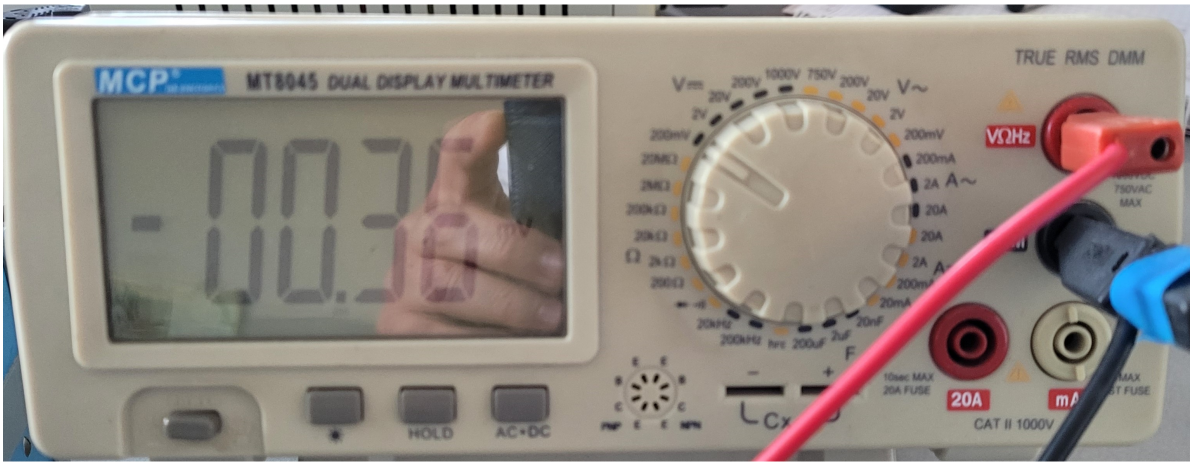


Figure 25.5. Multimeter MT8045 for measurement U_2 and calculate the photocurrent $I_{photo} = U_2/10\text{M}\Omega$.

2. Set the selected filter. The table 5.2 lists the optical filters with their central transmittance wavelength (with an accuracy of 2 nm) for which the maximum light transmittance of the filters is observed.
3. For a selected filter available on the workstation do the following: set U_1 voltage to 10 V and keep gradually lowering it while recording the U_2 value until it reaches zero;
4. Cut off the light; set $U_1 = 0$ and read U_2 which is the U_0;
5. Turn the light on again; set the negative voltage U_1 such that the previously measured U_2 has the reference value U_0; then the U_1 is the stopping potential V_h;
6. Determine the stopping potential V_h for all the remaining filters; do the standard series of three measurements of V_h for each filter.

Table 5.2. The list of optical filters with the wavelengths of their maximum transmittance (with an accuracy of 2 nm)

No filter:	1	2	3	4	5	6	7	8	9	10
Wavelenght (λ) [nm]:	400	425	436	500	550	575	600	625	650	675

Table 5.3. Accuracy of DC voltage measurement with the V562 meter - used for measure the U_1 voltage.

Range	Resolution	Precision
200 mV	0.1 mV	±(0.5% of reading + 0.1) mV
2 V	1 mV	±(0.5% of reading + 1.0) mV
20 V	10 mV	±(0.5% of reading + 10) mV
200 V	100 mV	±(0.5% of reading + 100) mV
2000 V	1000 mV	±(0.5% of reading + 1000) mV

Report:

1. Knowing values of U_2 and R calculate the photocurrent $I_f = \frac{U_2}{R}$ and plot it as a function: $I_f = f(U1)$;
2. Knowing the wave lengths of the filters – calculate the frequencies $\nu = \frac{c}{\lambda}$;
3. Plot the stopping potential V_h as a function of frequency $V_h = f(\nu)$,
4. By using linear regression: determine the slope coefficient a_{reg} and the point at which the function crosses Y axis b_{reg};
5. Calculate Planck's constant ($h = a_{reg}e$) and work function ($W = b_{reg}e$) and their standard deviations;
6. Write down the final conclusions

Keywords:

- Photoelectric phenomenon: macroscopic description, photocurrent lag in relation to illumination, photocurrent intensity and lighting, electron energy and lighting
- Microscopic description, Einstein equation, output work, cutoff frequency (wavelength)
- Photocell, braking voltage, method of determining Planck's constant and output operation
- Linear Regression

26. Determination of ferromagnetic hysteresis loop by means of a hallotron

Introduction

In five elements (Fe, Co, Ni, Gd and Dy) and in many compounds and alloys of these and other elements, there is a special effect that allows to obtain a high degree of magnetic order. In these metals and compounds, called *ferromagnets*, there is a special form of interaction called *the exchange interaction* that couples the magnetic moments of atoms together in a rigid-parallel fashion. This phenomenon occurs only below a certain critical temperature, the so-called *Curie temperature*. Above the Curie temperature, the exchange coupling disappears and the body becomes *paramagnetic*.

The presence of ferromagnetic material strongly influences the parameters of the magnetic field. Consider a ring-shaped ferromagnet with a toroidal coil wound over it. When a current of i_m flows through the coil without a ferromagnetic core, a magnetic field with an induction B_0 is created inside it:

$$B_0 = \mu_0 n i_m. \tag{26.1}$$

In the above formula, n denotes the number of turns per unit length of the toroid, μ_0 - magnetic permeability of the vacuum ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).

After entering the core toroid, the induction reaches B, which is many times greater than B_0. The reason for the increase in induction is the reordering of the elementary atomic dipoles in the core and the creation of its own magnetic field that adds to the external field. Therefore, the *total induction* can be expressed as:

$$B_0 = B_0 + B_M, \tag{26.2}$$

where B_y denotes the magnetic induction from the core. The B induction in the interior of the ferromagnetic can also be expressed as follows:

$$B = \mu\mu_0 n i_m, \tag{26.3}$$

where μ is a dimensionless quantity called the magnetic permeability of the medium, defining how many times B is greater than B_0. The dependence of the B induction on the magnetizing current is not linear, because in the case of μ ferromagnets it strongly depends on the magnetic field strength H. The magnetic field is proportional to the intensity of the magnetizing current:

$$H = n i_m. \tag{26.4}$$

The aforementioned ordering of magnetic moments does not apply to the entire material, but to certain areas called domains. Within the ferromagnetic domain, the magnetic dipoles are parallel to each other, regardless of external conditions, while the ordering directions in different domains are different. In the non-magnetized state, the menus are positioned completely randomly (with the order inside the domains preserved), and

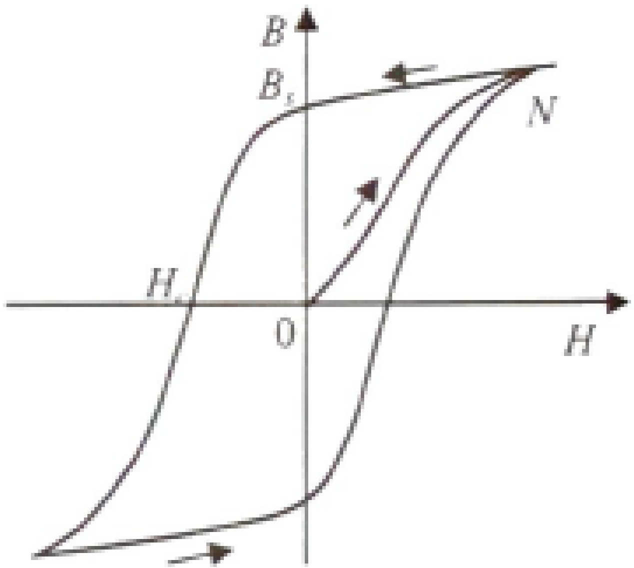


Figure 26.1. Ferromagnetic hysteresis loop; H - magnetic field strength, B - magnetic induction in the material, B_S, - magnetic residue, H_C - coercive field, N - saturation

magnetization consists in the orientation of more and more domains towards the external field.

For small and medium values of the magnetic field, the induction increases as a result of changes in the size of the domains and their subsequent rotation - in equation (26.2), the expression determining the increment of B is B_M.

After reaching saturation (ordering all dipoles) in a strong field, the value of B_M becomes stable, while B_0 continues to increase linearly.

The mechanism presented here describes the magnetization of a sample that was completely demagnetized in its initial state. The graphic image of this process is the so-called primary magnetization curve in the diagram $B = f(H)$ (line $0 - N$ in Fig. G.4).

After the maximum order is achieved, coupling forces also appear between the domains, which keeps order even after subtracting the external field. The value of magnetization at zero external field (but after previously achieved saturation) is called *magnetic residual* or *spontaneous magnetization*.

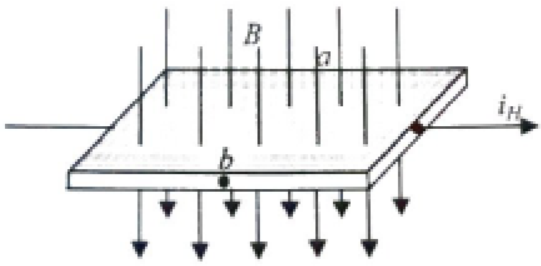
In order to cancel this magnetization, we have to apply an outer field of the opposite direction and with an appropriate value, called the *coercive field*. At this point, the magnetization is zero. If the field continues to grow in the same direction, the domains are inverted and the ordering process is repeated in the opposite direction.

Note that the induction B in the sample, as well as its magnetization, depend not only on the value of the H magnetizing field, but also on the "history of the sample", i.e. on its current state. The full course of the dependence of induction on the magnetic field strength is called *hysteresis loop*, its typical shape is shown in Fig. G.4.

Measurement

To measure the magnetic induction we use an iron ring in which a narrow slit is cut perpendicular to the induction line. The induction in the narrow slit differs little from the value inside the ferromagnetic.

We measure induction in the gap with a *hall effect sensor*. The basis of the hall effect is the *Hall effect*, consisting in the formation of the potential difference V_h between points a and b (Fig. 26.2) of a thin semiconductor plate or conductor as a result of the magnetic field acting on moving electric charges. The charges that make up the current z are in a magnetic field perpendicular to the direction of the current. In such a situation, the charges are influenced by the Lorentz force pushing them towards $a - b$, which causes a potential difference between these points.



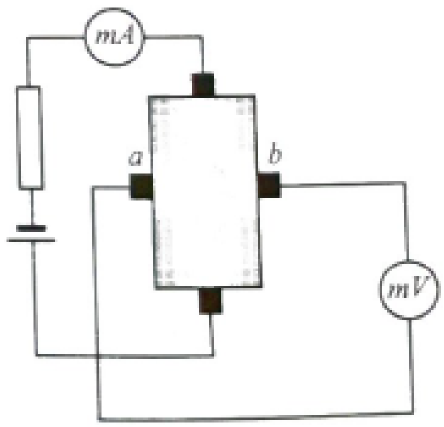


Figure 26.2. Hall effect sensor in a magnetic field (above) and a Hall voltage measurement system (next to it)

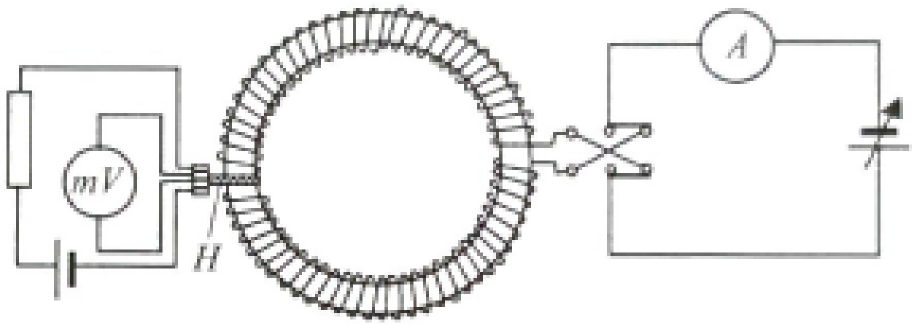


Figure 26.3. Hysteresis loop measurement system, H - hall effect sensor.

The potential difference V_h, also known as *the Hall voltage*, is proportional to the current flowing through the hall effect as well as to the magnetic induction, and depends on the type of material and the dimensions of the hall effect sensor.

$$V_H = \gamma i_H B, \tag{26.5}$$

The γ factor, called the sensitivity of the halothron, is determined by the individual properties of the instrument. Once the sensitivity is known, the measurement of the magnetic induction is reduced to measuring the Hall voltage and the hall effect current and using the equation (26.5). The ferromagnetic material, which is the subject of our research, has the shape of a toroidal ring with a cut-out slot enabling the placement of a halotron. A winding is wound on the ring through which a magnetizing current flows.

The measuring system to determine the hysteresis loop is shown in fig. 26.3. It consists of two parts. The first is the magnetizing winding power supply (to the right of the ring), which includes a DC source, an ammeter, and a current direction switch. The second part is the Hall voltage measurement circuit - it contains a hall effect sensor, a hall sensor and a millivoltmeter. The same system in terms of which it shows its functions more clearly is also shown in Fig. 26.2.

Measurements:

1. While changing the current i_m, record the corresponding Hall voltage V_h according go the following procedure;
2. Increase the current gradually (step of 0.2 A) up to ~3 A (not to exceed 3 A);
3. Decrease the current gradually to zero;
4. Switch the direction of current;
5. Repeat the measurements up to ~3 A and down to zero again;
6. Switch the direction of current and finish up the measurements reaching 3 A value once again.

Report:

1. Determine the magnetic field $H = n * i_m$ ($n = 6$/cm or 600/m, i.e. number of coil turns; i_m – magnetizing current) and the induction B knowing that $V_h = \gamma \cdot i_H \cdot B$, where
   ($\gamma = (140 \pm 5)$ V/AT; $i_H = (10,0 \pm 0,5)$ mA, V_h – measured Hall voltage);
2. Plot $B = f(H)$;
3. Determine ΔB and ΔH; Calculate the errors by applying the method of logarithmic or complete differential to equation (26.4) and to the transformed equation (26.5).
4. Mark error rectangles for several points on the plot.
5. Write down the final conclusions

Keywords:

- Ferromagnets, Curie temperature
- Magnetic induction in vacuum and in matter, magnetic permeability. Magnetic field intensity, units of intensity
- Ferromagnetic domains, microscopic image of magnetization
- Hysteresis loop, spontaneous magnetization, coercive field
- Hall effect, Lorentz force, Hall voltage, Hall effect, induction measurement

27. Calibration of the thermocouple

Introduction

The thermocouple consists of two different conductors connected to each other as shown in fig.27.1. If the points where the conductors connect have different temperatures, a potential difference is created between them, called the thermoelectric force. Its value depends on the type of conductors that make up the thermocouple and on the temperature difference and is expressed by the formula:

$$\varepsilon = \alpha_1(T - T_0) + \alpha_2(T - T_0)^2, \tag{27.1}$$

where α_1 and α_2 are the thermoelectric coefficients that characterize the materials used. This effect is called the *Seebeck effect.*

The direct cause of the thermoelectric force is the different value of contact voltages in joints with different temperatures. We will understand the existence of *contact voltages* and their dependence on temperature by considering electron phenomena in metals.

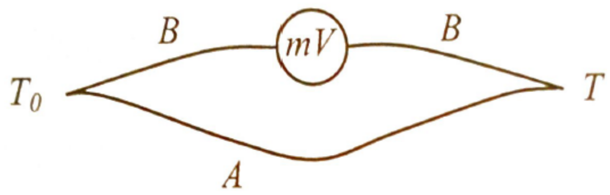


Figure 27.1. Thermocouple

Figure 27.2 shows the occupied electron levels in two different conductors - A and B. Their Fermi levels E_F lie at different distances from the vacuum level E_0, so the work function W_A and W_B are also different. At each temperature, there are a number of electrons that have sufficient kinetic energy to do the work of W exit, that is, to go beyond metal surface. These electrons create the so-called thermocouple current directed perpendicular to the metal surface. Density of the thermocouple current is determined by the Richardson-Dushman law and for both conductors it is respectively:

$$j_A = AT^2 e^{\frac{W_A}{kT}}, \tag{27.2}$$

$$j_B = AT^2 e^{\frac{W_B}{kT}}. \tag{27.3}$$

When both conductors are brought very close together, electrons leaving metal A will go to metal B and vice versa. In the situation presented in Fig. 27.2, $j_A > j_B$ due to the values of the work functions ($W_A < W_B$). The advantage of the j_A current leads to an increase in the number of electrons in the B metal and to their shortage in the A metal.

In this situation, the metals will be charged with opposite signs and a potential difference will arise between them in such a direction that the further flow of electrons from A to B will be impeded and will eventually be balanced by the flow from B to A. In the equilibrium state, presented in Fig. 27.2b, the electrons flux in both directions are the same, which means that the power exponents in equations (27.3) and (27.3) are equal, i.e.

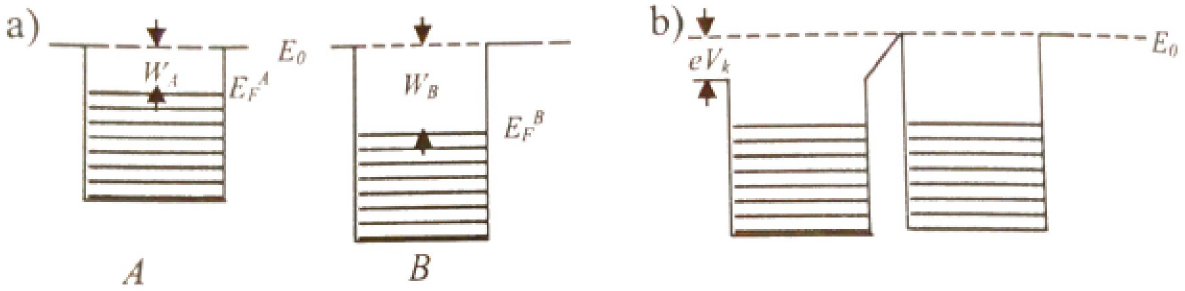


Figure 27.2. Energy bands of two separate conductors (a) and closely connected (b)

$$W_A + eV_k = W_B. \tag{27.4}$$

The above equation reflects the fact that the electrons leaving the metal A must do, in addition to the output work, work against the potential difference V_k. This difference of potentials resulting from the contact of two conductors is called *contact voltage*. Its value is determined only by the difference in the work function of both metals.

$$V_k = \frac{W_B - W_A}{e}, \tag{27.5}$$

where e is the charge of the electron.

When both connectors of the circuit in Fig. 27.1 are at the same temperature, their contact voltages compensate each other and the resultant voltage is zero. Similarly, in any closed circuit composed of more than one conductor, the sum of the contact voltages is equal to zero.

Contact voltage changes with temperature. This is due to the dependence of Fermi energy on temperature. This relationship is described by the equation:

$$E_F = E_{F0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right], \tag{27.6}$$

where E_{F0} is the Fermi energy at 0 K.

When the temperature of a given contact changes, the changes in the Fermi energy result in a different value of the work function of both metals, which leads to a change in contact voltage. Thus, only with a temperature difference of the joints in the circuit will there be a resultant voltage called the thermoelectric force.

Thermoelectric force can also arise in a homogeneous conductor (without connectors), when we create a temperature difference between its ends. This phenomenon is known as *the Thomson effect* and is a simple consequence of the Fermi energy dependence on temperature.

If we apply external voltage to a circuit containing connectors of different conductors, electric current will flow. When a junction appears on the path of the current, where the contact potential decreases in the direction of the current, heat is released at the junction, according to the Joule-Lenz law $Q = iV_k t$. When the contact potential drops in the opposite direction to the current, cooling of the junction occurs. We call *the Peltier effect* the taking or the release of heat when electricity flows through metal junctions.

Thermoelectric phenomena are now often used both for measuring temperature in a very wide range and for detecting very little heating. Thermocouples, also known as thermocouples or thermocouples, are used to measure a temperature that is not too low. Measurement thermocouples consist of conductors with a known, previously well-measured thermoelectric voltage. At the point of contact, the conductors, most often in the form of wire, are welded or soldered. One of the contacts (fig. 27.3) is inserted in a medium with a specific temperature of T_0, e.g. in a mixture of ice and water, and another in the place where the temperature of T is to be measured. The voltage generated in the circuit is measured with a millivoltmeter. On the basis of the measured voltage we determine the difference $T - T_0$ and then the temperature T itself.

To measure the temperature from −200°C to +350°C we use copper-constantane thermocouples, in the range of $0 \div 1000$°C iron-constantane, and to high temperature measurement, prevailing in laboratory and industrial furnaces (up to 1700°C are thermocouples in which one wire is made of pure platinum, and the other is made of a 90% platinum and 10% rhodium alloy.

In technical applications, a simplified version of the thermocouple is used, presented in Fig. 27.4. One connector is placed in the test center, and the voltage meter is switched on in place of the other. The meter readings correspond to the temperature difference between the medium and its surroundings. Compared to liquid thermometers, thermocouples have the following advantages:

- have a very low heat capacity; they can be made of even the thinnest wires, making them suitable for temperature measurements of micro-objects
- measurement sites may be located at large distances from the indicator;
- have a very large range of measured temperature from −250 to 2000°C.

Thermocouple calibration

In order to find the thermoelectric voltages corresponding to the specified temperature differences $T - T_0$ we use the system shown in Fig. 27.3 or its variants. One connector is in a vessel containing a mixture of ice water ($T_0 = 0$°C), and the other in an environment whose temperature can be regulated. It can be a vessel with water, the temperature of which is changed with a heater, as well as a special electric heater surrounding the thermocouple junction. In all cases, the temperature is measured with a thermometer. Due to the inertia of the thermometer, the temperature rise cannot be too fast. Adjustable temperature rise can be achieved by using an autotransformer or a power supply that regulates the heater voltage. After each voltage change, wait for the temperature reading to stabilize.

After finding the thermoelectric voltages for different temperature values, we plot the graph. If it is a straight line, then α_2 in equation (27.1) is zero and α_1 is the slope of the line.

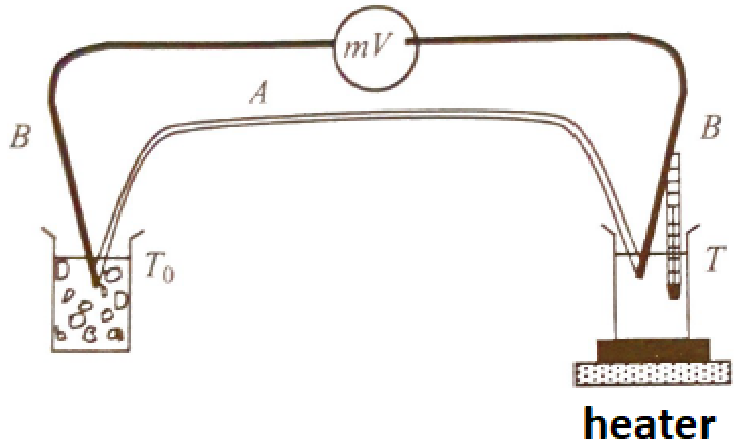


Figure 27.3. Thermocouple gauge system; A, B - various conductors.

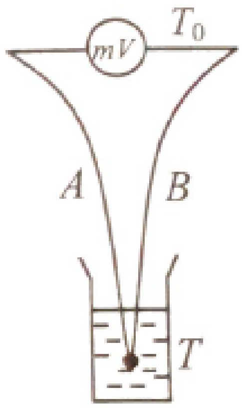


Figure 27.4. Technical thermocouple; A, B - various conductors.

Measurements and Report:

1. Assemble the measuring system as shown in Fig. 27.3.
2. Find the zero reading by shorting the terminals of the millivoltmeter.
3. By gradually heating the thermocouple junction, measure the temperature approximately every 5°C and the corresponding thermoelectric voltage.
4. Measure the thermoelectric voltage on each thermocouple (A to C) as a function of temperature – according to the instructions given in class.
5. Make similar measurements while cooling down.
6. Plot the dependence of thermoelectric voltage on temperature ($U = f(Temp.)$) for each thermocouple.
7. If the plot points are in a straight line, find the thermoelectric α_1 using linear regression. If the graph is clearly non-linear, determine the factor separately for the beginning and end of the range. To do this, apply a linear regression twice to only a few measuring points, respectively start and end.
8. Find the slope coefficient errors.
9. Round off the results and errors and make a final statement.
10. Write down the final conclusions

Keywords:

- thermocouples, Seebeck effect,
- Energy levels in metals, Fermi level, output work
- Thermoemission, Richardson-Dushman law
- Contact voltage, thermoelectric force
- Thomson phenomenon, Peltier phenomenon
- Construction of thermocouples, temperature measurement, advantages of thermocouples

28. Measurement of the e/m ratio by means of deviations in the magnetic field

Introduction

A force called the Lorentz force, determined by the formula, acts on an electrically charged particle moving in an electric and magnetic field:

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}), \tag{28.1}$$

where: q - particle charge, v - its speed, E - electric field strength, B - magnetic induction. In general, the action of both of these fields leads to a change in the velocity vector, in the electric field the direction of velocity may change, while in the magnetic field the velocity value remains constant and the direction changes.

The study of the behavior of charged particles, such as electrons, positive ions, in electric and magnetic fields allows to determine the specific charge, i.e. the ratio q/m.

To determine the specific charge of the electron (e/m) we will use an oscilloscope tube with a magnetic deflection in the direction of Y. The structure of such a Lamp is shown in Fig. 28.1. The electrons emitted from the heated cathode as a result of the phenomenon of thermo-emission are then accelerated due to the difference of the U_A potentials between the cathode and the anode A. In order to focus the electron beam, the anode is usually in the form of several cylinders with appropriate potentials. Then the electrons pass between the CC plates, which are usually used for horizontal deflection, but will not be used in our exercise. A little further, the electrons enter the area of the magnetic field directed horizontally, perpendicular to the direction of their movement. According to the vector equation (28.1), the electrons are deflected vertically (Fig. 28.2). After exiting the magnetic field, they run in a straight line and eventually hit the fluorescent screen, causing it to glow.

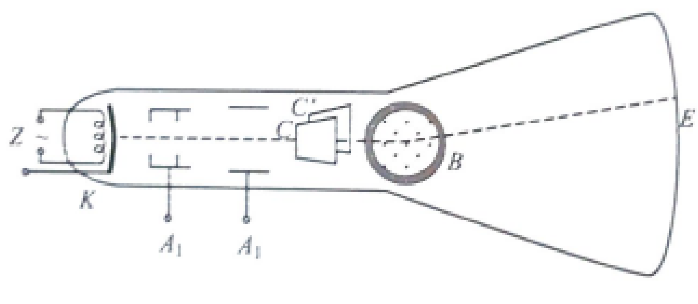


Figure 28.1. Construction of an oscilloscope tube, Z - cathode incandescence (K), A - anodes, B - magnetic field generating coils (one in front of and one behind the tube), C - electrostatic deflection plates , E - fluorescent screen

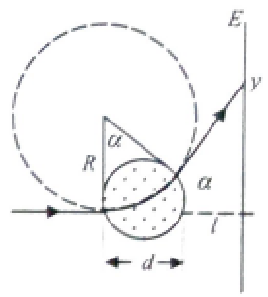


Figure 28.2. In the magnetic field, (the dotted area), the electron moves along an arc with the radius of curvature R

The magnetic field is created by the flow of current through two coils placed opposite each other outside the lamp. The magnetic induction B in the area between the coils has a direction parallel to the axis of the coils and its value B is proportional to the current strength I:

$$B = cI. \tag{28.2}$$

Let's find an expression that allows us to find the ratio e/m from the position of the light spot on the screen. First note that the directions of electron velocity and magnetic induction are perpendicular to each other and that in this case the value of the deflecting force in the deflection in the magnetic field can be scalar expressed as evB. This force causes the electron to move along the arc and the appearance of a force of inertia (centrifugal) that opposes the contraction of the arc. Shortly after the electron exits the magnetic field, both forces will balance and the electron will move around the circle. The equation of both forces is expressed by the equation:

$$evB = \frac{mv^2}{R}, \tag{28.3}$$

where R is the radius of curvature of the track. The searched quantity e/m based on the equation (28.3) can be represented as:

$$\frac{e}{m} = \frac{v}{BR}, \tag{28.4}$$

We can express the speed by the voltage U_A, equating the kinetic energy to the work done by the electric field on the path between the cathode and the anode:

$$\frac{mv^2}{2} = eU_A. \tag{28.5}$$

We insert the speed calculated from the above equation into equation (28.4), we square both sides and we get:

$$\frac{e}{m} = \frac{2U_A}{B^2R^2}, \tag{28.6}$$

There is only one amount left to eliminate - R. Taking into account that in the conditions of the experiment $y << l$ and $d << R$ (Fig. 28.2), we can write:

$$\alpha = \frac{y}{l} = \frac{d}{R}. \tag{28.7}$$

The radius of curvature R can therefore be expressed as:

$$R = \frac{ld}{y}. \tag{28.8}$$

where: l - the distance of the oscilloscope tube screen from the center of the coil, d - diameter of the deflecting coil, y - the deviation of the spot on the screen relative to the position at $B = 0$.

After inserting formulas (28.2) and (28.8) into equation (28.6), we get the final expression from which we can calculate the ratio e/m we are looking for based on simple measurements:

$$\frac{e}{m} = C\frac{y^2}{I^2}, \tag{28.9}$$

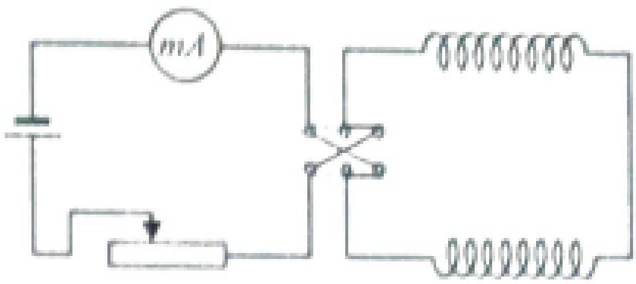


Figure 28.3. Power supply diagram of the deflecting coil.

The constant $C = 2U_A/(c^2l^2d^2)$ includes only the quantities characterizing the apparatus.

Measurements:

1. Read the spot position at zero coil current (y_0).
2. By changing the coil current approximately every 5 mA in the range from 20 mA to 110 mA read the position of the spot on the screen. Repeat the measurements for the opposite direction of the current.
3. Note the accuracy of the spot position reading and the accuracy of the measurement of the current flowing in the circuit.

Report:

1. Using the obtained data, determine the spot deviation in each measurement.
2. Determine the e/m_e ratio for each calculated spot deviation.
   $\frac{e}{m_e} = C\frac{y^2}{I^2}$, where $C = \frac{2U_A}{c^2l^2d^2} = (7.7 \pm 0.1) \cdot 10^{11}$ $\mathrm{A^3skg^{-1}m^{-2}}$, I [mA] - current, y [mm] - deviation of the spot on the screen relative to the position for $B = 0$, $e = 1.602176634 \times 10^{-19}$ C (exact) - charge of electron, $m_e = 9.1093837015(28) \times 10^{-31}$ kg - mass of electron, l - distance of the oscilloscope lamp screen from the center of the coil, d - diameter of the deflection coil, c - proportionality factor between magnetic induction B and current I ($B = cI$).
3. Calculate the average value from the results obtained. Specify the accuracy of the result.
4. Present the final results of the experiment (properly rounded).
5. Write down the final conclusions. Compare the result with literature data.
   [9] CODATA 2018 ($-e/m_e = -1.75882001076(53) \times 10^{11}$ C kg^{-1})

Keywords:

- Electron charge and mass, charge in electric and magnetic field, Lorentz force,
- Oscilloscope lamp, charge speed obtained in electric field,
- Coil magnetic field, magnetic field path and force balance,
- Calculations e/m, quantities to be measured.

6. Optics

29. Determination of the refractive index of apparent and real thickness of the plates

Introduction

The light observed in everyday life sometimes passes through one or more mediums. This causes the observer to get the impression that the light is coming out from a different point than it actually is. This phenomenon is called the image of the real source or *virtual source.*

They look at the objects lying on the bottom of the vessel with water, it seems to us that they are lying closer to the surface than in reality. On the contrary, a diver looking up at, say, a hanging branch of a tree will think it is higher than it actually is. In both cases there is an apparent change in distance resulting from the refraction of light at the border of two mediums.

The physical basis of the method

An example of the phenomenon in which there is an apparent change in thickness, as well as the principle of measuring this thickness is shown in Figure 29.1.

The rays reflected from the lower surface (point C) refract when they reach the upper surface. To understand this phenomenon, Snell's law should be used, which is as follows at the interface of air-medium / two mediums of different refrractive indexes:

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}, \tag{29.1}$$

where α is the angle of incidence, β is refraction angle, n_2 value of the refractive index of the medium, and n_1 the value of the air's refractive index (for vacuum $n_1 = 1$, for air $n_1 \approx 1$). After substitution $n_1 = 1$ we get:

$$\frac{\sin\alpha}{\sin\beta} = n, \tag{29.2}$$

where n is absolute refractive index of any medium.

In order to calculate the apparent thickness h, we assume that the incident rays on the surface on the plate form a very small angle with the line perpendicular to the surface at the point of incidence. When we look vertically down at the plate, we can

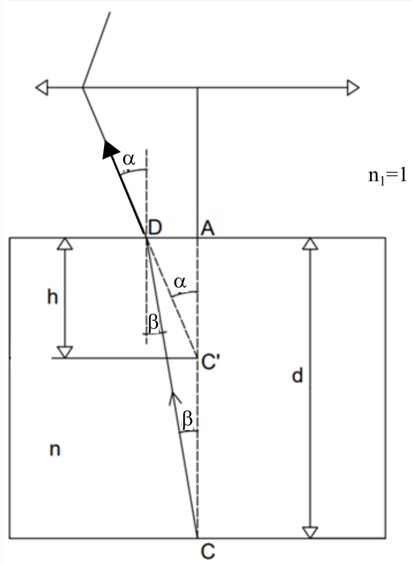


Figure 29.1. Position of the microscope lens (focusing lens) relative to the plate to observe its upper and lower surface.

assume that α and β are very small ($\alpha \to 0$ and $\beta \to 0$, so $\cos\alpha \approx 1$ and $\cos\beta \approx 1$) therefore:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \approx \sin\alpha \qquad \text{and} \qquad \tan\beta = \frac{\sin\beta}{\cos\beta} \approx \sin\beta. \tag{29.3}$$

From the figure you can see that:

$$\tan\alpha = \frac{AD}{h} \approx \sin\alpha \qquad \text{and} \qquad \tan\beta = \frac{AD}{d} \approx \sin\beta. \tag{29.4}$$

After substituting the above values in the equation 29.2, we obtain the relationship between the apparent thickness h and the actual d:

$$n = \frac{d}{h}. \tag{29.5}$$

The above relationship allows the calculation of the refractive index based on d and h measurements.

Measurement Principle

The real thickness of the sample made of glass is measured with a micrometer screw, and then the apparent thickness is measured with a microscope. The subject of the study will be scratches on the upper and lower surfaces. First, we find a sharp image of the upper scratch on the computer monitor screen, we read the position of the microscope table on the micrometer (a_u). Then we look for a sharp image of the lower scratch and read the position of the microscope table on the micrometer (a_l). The apparent thickness of the tile can be determined from the formula:

$$h = a_l - a_u. \tag{29.6}$$

The microscope we use must be able to move the table or tube. Most microscopes have scaled rotations of the precision tube knob. Using the fine knob, we scale the position from the top to the bottom of the crack, or vice versa, because the coarse feed knob is not usually scaled. At the beginning, using the coarse feed, you should set the visual acuity of the bottom scratch (it is recommended to set them on e.g. the bottom surface of the tile, and then set the image on the bottom scratch). In this position, we do not move the coarse knob anymore, and with the help of the precision knob we set a clear image on the upper scratch. To calculate the tube travel, the number of full revolutions should be taken into account, as well as the differences in the knob indication.

Measurements:

1. Turn on the computer and start the AMCAP program.
2. Turn on the microscope backlight.
3. Using a micrometer screw, measure the thickness d of the plate. Repeat the measurement at least ten times. Note the measurement uncertainty (Δd).
4. Place the plate on the microscope table, so that the cross between the scratches on the plate is just below the lens.
5. By changing the position of the table with a tile, find the position a_u, in which the upper scratch is clearly visible, and then the position a_l, in which the lower scratch is clearly visible.
6. Repeat the measurement ten times, each time "spoiling" the image and looking for it again.
7. Note the measurement accuracy Δa_u and Δa_l.
8. Repeat the measurement for subsequent plates available on the test bench.

Report:

1. Determine the average values of actual (d) and apparent ($h = a_l - a_u$) thickness from the obtained data. Then calculate the uncertainty of both thicknesses.
2. Determine the refractive index $n = \frac{d}{h}$ and its uncertainty Δn.
3. Present the final results of the experiment (properly rounded).
4. Write down the final conclusions

Keywords:

- Snell's law, refractive law, refractive index, light passage through a parallelepipedal plate,
- course of rays from the upper and lower surfaces to the lens, observation of the plate by a microscope, real and apparent thickness,
- expression of the refractive index by real and apparent thickness,
- method of measuring actual and apparent thickness

30. Determination of focal length lenses from a lens pattern and the Bessel method

Introduction

A lens is a transparent body limited by two spherical surfaces. The axis connecting the centers of curvature of both surfaces is called the *optical axis* of the lens. The light passing through the lens is refracted successively on both of its surfaces (in the drawings, for convenience, a single refraction is usually marked on the so-called middle surface). The ray passing through the *optical center* of the lens is not refracted regardless of the angle of incidence on the lens - it undergoes only a slight parallel shift.

The beam of rays that runs parallel to the optical axis after passing through the lens focuses on one point called the *focal point* (Fig. 30.1). The distance of the focus from the center of the lens is called the *focal length*.

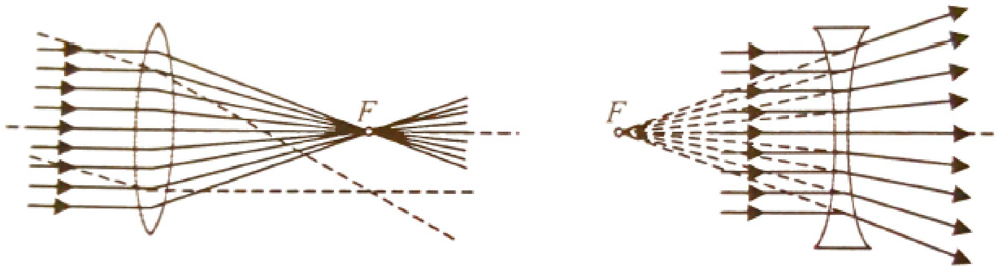


Figure 30.1. Focusing rays in the focus of a converging lens (left) and a diffusing lens (right)

By choosing the right radius of curvature, you build *focusing* and *diffusing lenses*. The parallel beam incident on the scattering lens becomes a divergent beam after passing through it. In this case, the focus is the point of intersection of the extensions of the refracted rays.

The position of the focal point depends on the refractive index n of the lens material with respect to the medium in which it is located, and on the *radii of curvature* of both the bounding surfaces R_1 and R_2. The dependence of the focal length f on the above parameters is given by the equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{30.1}$$

The reciprocal of the focal length is called the focusing ability of the D lens:

$$D = \frac{1}{f}. \tag{30.2}$$

The unit of the focusing power is the diopter of m^{-1}.

The lenses have the ability to map points in that the rays coming from the point O, called the object, are focused after passing through the lens at the point I, creating an image of the object. The position of the image depends on the position of the object and the focal length of the lens - the so-called lenticular equation

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}, \tag{30.3}$$

where o is the object distance from the lens, i - the image distance from the lens.

The equation 30.3 can be used when:

- rays extending from O create a small angle with the optical axis,
- the lens is thin, i.e. its thickness is small compared to the radii of curvature.

In relation to the distances o, *and*, R_1, R_2 and f, the following characters are valid:

- o is always positive,
- i, R and f are positive when it lies on the opposite side of the lens to the object,
- R and f are negative when on the same side of the lens as the object.

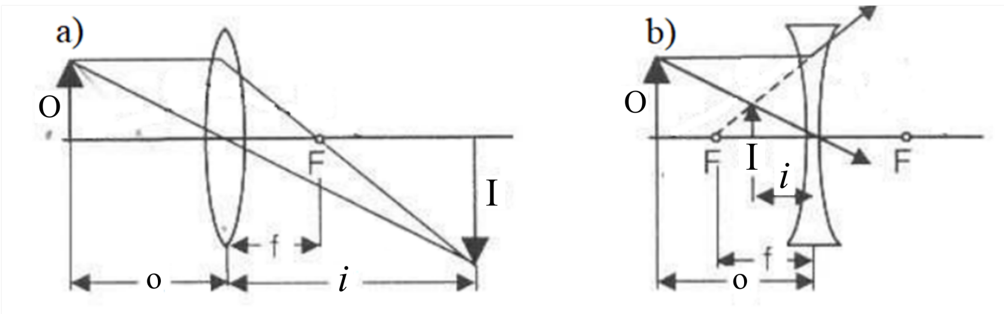


Figure 30.2. The image construction by the focusing lens (on left) and diffuse lens (on right): O - object, I - image, F - lens focus.

We can find the image in the lens using the geometric construction shown in Fig. 30.2. In the construction of the image, we use two characteristic rays:

- a ray parallel to the optical axis which, when refracted, passes through the focus,
- the ray passing through the optical center does not change direction.

The image is created at the point of intersection of these rays or their extensions.

Linear zoom is the ratio of the size of an image to the size of an object. It is also equal to the ratio of the distances i and o

$$m = -\frac{i}{o}. \tag{30.4}$$

The "-" sign has been introduced so that the magnification is positive when the image is straight and negative when it is inverted. An illustration of the equation (30.3) is the graph in Fig. 30.3 showing the dependence of the distance of the image on the distance of the object from the lens. The diagram shows hyperbolas, the individual

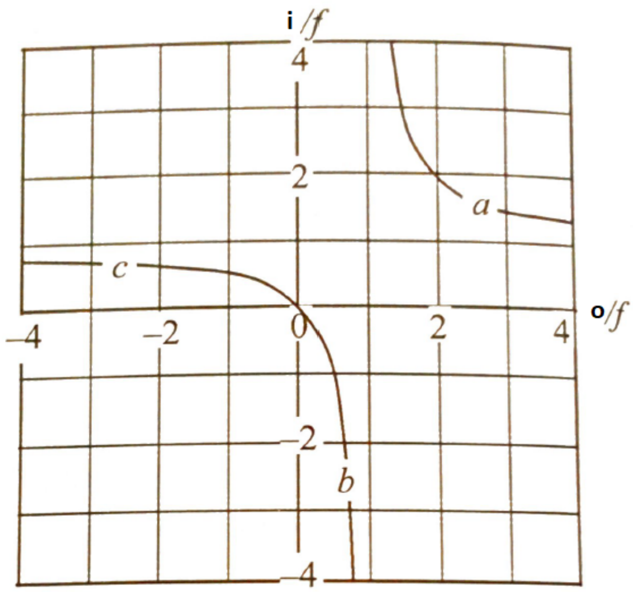


Figure 30.3. Lenticular equation plot; a - focusing lens, real image inverted, b - focusing lens, simple imaginary image, c - scattering lens, simple imaginary image

parts of which correspond to the situations marked in the figure. From Figure 30.3, you can easily find all the features of an image depending on the position of the object.

The focal length of a system consisting of two thin lenses f_1 and f_2, located at a distance of d from each other, is given by the formula:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}. \tag{30.5}$$

Methods of finding focal lengths

Based on the lens pattern

The distances o and i occurring in the formula (30.3) are easily measurable, so we can use this formula to determine the focal length f. We place a glowing object, a lens and a screen on the optical bench in such a way as to obtain a clear image of the object on the screen. The screen and lens frame are placed on trolleys, which allows them to slide along the bench. The wheelchair pointer moving relative to the scale on the bench marks the exact position of the wheelchair or lens.

If the lens carriage does not have an indicator or the indicator is offset from the center of the lens, then:

- we read the position a_1 of any edge of the trolley,
- then we rotate the cart or the lens frame by 180 ° and read the position a_2 of the same point of the cart as before,
- we calculate the real position of the lens a as the arithmetic mean of both positions.

Since the estimation of the sharpness of the image is associated with high uncertainty, we repeat the position of the trolley several times, note the position of the indicator each time, and then calculate the average value.

Knowing the appropriate positions, we calculate the distance of the image and the object from the lens, and then from equation (30.3) we find the focal length.

The described method cannot be applied directly to diffusing lenses as they do not give a real image. However, we can calculate the focal length of the system consisting of the tested diffusing lens and the converging lens. Given the focal length f and the focal length f_c of the converging lens, we find the focal length f_d of the dispersing lens from equation (30.5). When using this method, it should be remembered that the real image will be obtained when the condition $|f_c| < |f_d|$ is met, and that the focal length of the diffusing lens is negative.

The Bessel method

The distances of the image and the object appear symmetrically in equation (30.3) - it means that after changing their values the equation still remains true. The physical consequence of the symmetry of the lens equation is the possibility of obtaining a sharp image with two positions of the lens relative to the object (Fig. 30.4).

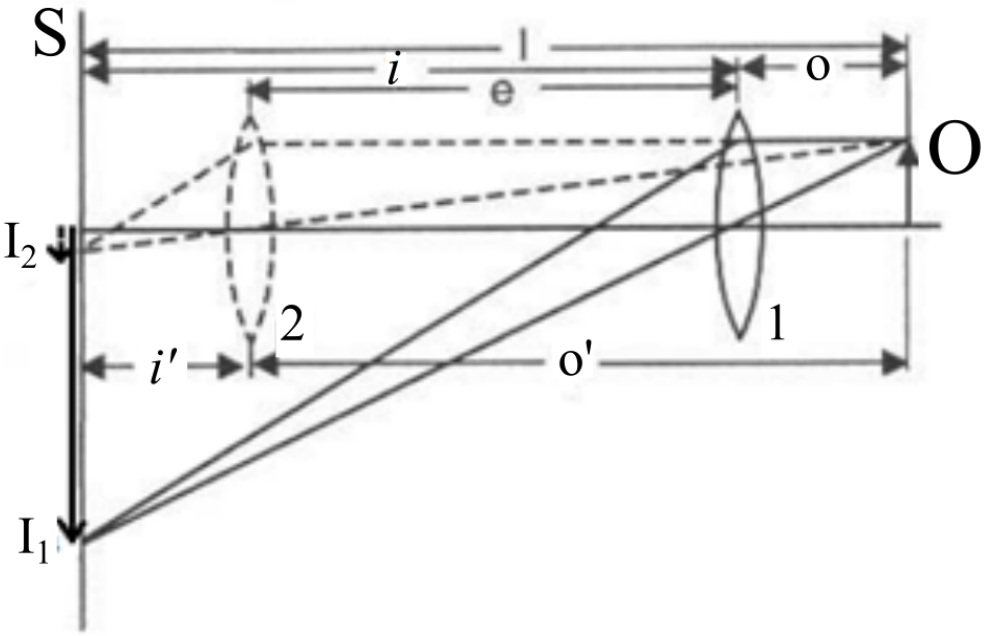


Figure 30.4. Illustration of the Bessel method for determining the focal length of the lens

For a constant distance l of the object from the screen, the image is created at the distance i and $i' = o$ from the lens. In one position the image is reduced in size and in the other it is larger in relation to the object. Based on fig. 30.4 and the previous paragraph, we can write a system of equations:

$$i + o = l,$$
$$i - o = e.$$

We calculate o and i from them and then insert the obtained values into equation (30.3). After simple transformations we get:

$$f = \frac{l^2 - e^2}{4l}. \tag{30.6}$$

To determine the focal length of the lens or lens system from equation (30.6), measure the object distance from the screen l and the distance e between the two lens positions at which the image on the screen is in focus. When looking for the e distance, we do not have to care about the exact position of the lens, because the difference in positions of the lens is equal to the difference in positions of any indicator associated with the lens holder. We make measurements for several screen positions.

Measurements:

1. Place the tested lenses in the holder.
2. Note the position of the luminous object on the scale attached to the optical bench (x_{obj})
3. Set the screen on the optical bench. Note its position (x_s).
4. Find the lens positions (two) so that the image on the screen is clear. Note them (x_1, x_2).
5. Reposition the screen several times and find clear images of the item again.
6. Take measurements for each focusing lens and for lens systems (each scattering lens with focusing lens). Note the distance between the lenses in the holder.

Report:

Determination of the focal length from the lens pattern

1. Determine the distance from each measurement:
   the object from the lens ($o = x_1 - x_{obj}$ or $o = x_2 - x_{obj}$)
   and the image from the lens ($i = x_s - x_1$ or $i = x_s - x_2$).
2. Calculate, using the lens formula $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$, focal lengths f of lenses obtained from individual measurements.
3. Calculate the average focal length for each lens f_{avg}. Determine it uncertainty $\Delta f_{avg} = 3 \cdot \sigma_s \cdot t_n$, where σ_s - standard deviation of average value, t_n - the Student-Fisher coefficient.
4. Find focal dispersing lenses from equation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$, where f - is focal of lenses set, f_1 - focal of focused lens, f_2 - focal of dispersing lens, d - distance between the lenses. Determine their uncertainty.

Determination of focal length by the Bessel method

1. Determine the distance from each measurement: the object from the screen and between the two lens settings for a clear picture.
2. Calculate, using the Bessel formula ($i + o = l$ and $i - o = e$ or $e = x_2 - x_1$ then $f = \frac{l^2 - e^2}{4l}$, focal lengths of lenses obtained from single measurements.

3. Calculate the average focal length and its uncertainty for each lens.
4. Find focal dispersing lenses. Determine their uncertainty.

Additional

1. Present the final results of the experiment (properly rounded). Compare both methods.
2. Write down the final conclusions

Keywords:

- thick and thin lenses, optical axis, optical center of the lens, refraction of the lenses,
- focus, focal length, focusing ability, diopter,
- lens equation, conditions of using the lens equation, signs of the size occurring in the lens equation,
- image construction, magnification,
- Bessel method

31. Determination of the diffraction grating constant

Introduction

Light is an electromagnetic wave, i.e. a wave consisting in the propagation of changes in the intensity of the electric and magnetic field in space. The decisive role in optical phenomena is played by the electric field intensity vector $\vec{E}$, also called the electric vector. Therefore, to describe the light wave it is enough to define this vector as a function of time and spatial coordinates. The behavior of the electric vector of a wave along the axis x is described by the wave function:

$$E = E_0 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \varphi\right], \tag{31.1}$$

where T and λ are period and wavelength respectively, φ_0 is the initial phase.

All kinds of waves, including light waves, can diffract and interfere. Huygens's principle is the basis for explaining these phenomena: each point reached by a wave becomes the source of a new spherical wave. Interference is the overlap of two or more waves. At a certain point in space, the amplitude will be amplified or weakened, depending on the phase difference of the overlapping waves. If two waves extend from points with the same initial phase, e.g. from different slots of the diffraction grating, there is a phase difference at the point of overlap due to the difference in traveled paths. The interference conditions can be expressed both by the phase difference $\Delta\varphi$ and by the path difference ΔS:

$$\circ\text{m}aximum: \quad \Delta\varphi = k \cdot 2\pi, \quad \Delta S = k\lambda, \quad k = 0, 1, 2, 3, ... \tag{31.2}$$

$$\circ\text{m}inimum: \quad \Delta\varphi = (2k+1) \cdot \pi, \quad \Delta S = (k + \frac{1}{2})\lambda, \quad k = 0, 1, 2, 3, ... \tag{31.3}$$

Although the interference occurs for any waves, a time-constant interference pattern can only be observed when coherent waves are superimposed, the phase difference of which does not change with time.

Single slit

We observe the *diffraction* of light as it passes through a small hole in an opaque obstruction. The essence of the diffraction phenomenon is shown in Fig. 31.1. Behind the opening, which is often a narrow slit, the behavior of the wave depends on the size of the opening in relation to the wavelength. When $a >> \lambda$, the width of the passing beam is basically equal to the width of the opening - the illumination of the screen parallel to the obstacle is the geometric image of the opening. In this situation, diffraction is not visible.

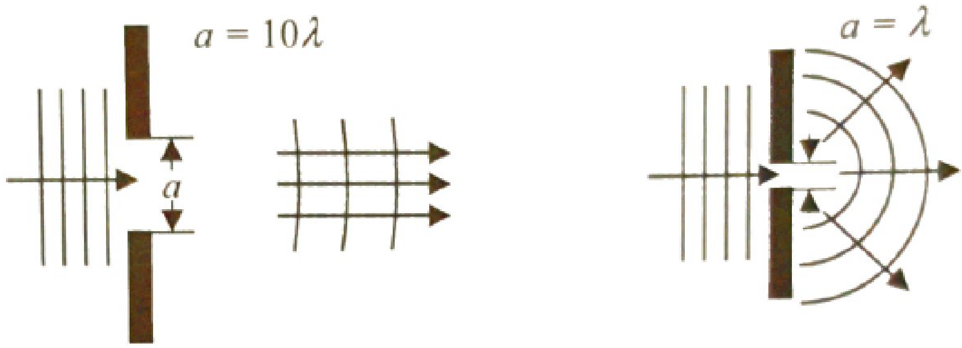


Figure 31.1. The light passing through openings of different sizes

This is not the case when $a \leq \lambda$; then the wave behind the slit is clearly spherical and illuminates the screen surface many times larger than the slit surface. It is a case of wave diffraction (deflection) on a single slit.

The diffraction image obtained on the screen is generally a system of wide fringes, alternating light and dark; it is the result of superposition of elementary waves coming from different fragments of the slit. The central maximum occurs on the extension of the direction of the incident waves, i.e. for the angle $\vartheta = 0$ (see Fig. 31.2), while the location of the successive *diffraction minima* (dark fringes) is determined by the relationship:

$$a \sin \vartheta = m\lambda. \tag{31.4}$$

Approximately halfway between adjacent minima there are the illumination maxima.

The width of the central maximum is determined by the location of the first minimum ($m = 1$). Formula (31.4) shows that for wide slits $a >> \lambda$ the first minimum appears at a very small angle, which means that the central maximum is narrow and reflects the geometric shape of the slit. When the slit width is equal to the wavelength, the first minimum occurs for the angle $\vartheta = 90°$, which means that the central maximum fills the entire space behind the slit. If the screen in this case is not too big, we can assume that its illumination is homogeneous.

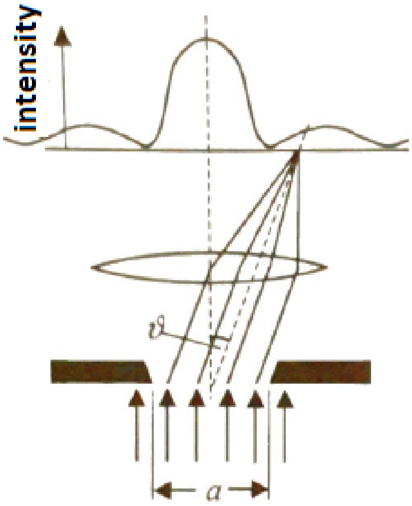


Figure 31.2. Single-slit diffraction

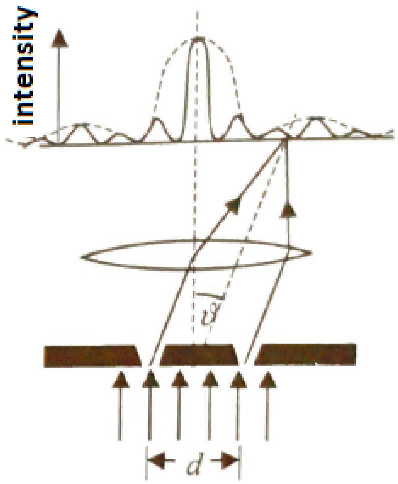


Figure 31.3. Diffraction on two slits

Two Slits

The image obtained on the screen when light passes through two slits (Fig. 31.3) is the result of the simultaneous occurrence of two phenomena: diffraction of light on each of the slits and interference of waves coming from adjacent slits.

Interference maxima occur at screen points for which the path difference ΔS is a multiple of the wavelength. From Figure 31.3 it can be seen that $\Delta S = d \sin \vartheta$, so the position of the interference maxima is determined by the relationship:

$$d \sin \vartheta = m\lambda, \quad m = 1, 2, 3, ... \tag{31.5}$$

The angular distance of the interference fringes is determined by the ratio λ/d where d is the distance between the centers of adjacent slits. *The relative intensity* of these fringes is determined by the diffraction pattern of a single slit and thus depends on the ratio λ/a, where a is the slit width. It can be said that the interference fringes are intensity modulated by the diffractive envelope. When the slots are very narrow, the diffraction pattern is very wide - all the interference fringes have almost the same intensity and only the interference image is visible on the screen.

The diffraction grating

Phenomena similar to those described above occur when the number of slots is greater. Such a system of parallel slits, situated at equal distances, is called *a diffraction grating*. Diffraction gratings are made by cutting grooves on the glass or on a metal plate with a diamond blade. Glass meshes are called *transmissive* because we observe the light after passing through the slots, while the metal meshes are called *reflective* ones, because the reflected rays are interfered with. Having prepared such a model diffraction grating, further grids can be made. For this purpose, the standard mesh is covered with a collodion solution, and then the hardened coating is removed and glued to a glass plate or other pad. The less accurate meshes are made by the photographic method.

In diffraction gratings, the width of the slits is on the order of the wavelength, so the intensity of the interference fringes is almost constant.

Increasing the number of slots from two to N does not change the position of the interference maxima, which are further described in equation (31.5), but causes some changes in their shape. Namely, with an increase in the number of fractures, the maxima become narrower. The angular width of the maximum is given by the formula:

$$\Delta\vartheta_0 = \frac{\lambda}{Nd\cos\vartheta_m}, \tag{31.6}$$

where ϑ_m, means the angle of occurrence of the maximum of the order m.

From equation (31.5) it can be seen that the position of the maximum changes with the wavelength. This property allows the use of diffraction gratings for spatial separation of the components of complex light, i.e. for spectral analysis. In order to be able to distinguish between wavelengths with little difference, the principal maxima should be as narrow as possible. We define the *resolving power* of a diffraction grating as follows:

$$R = \frac{\lambda}{\Delta\lambda}, \tag{31.7}$$

where λ is the average wavelength of the two spectral lines barely distinguishable and $\Delta\lambda$ is the wavelength difference between them.

According to *the Rayleigh criterion*, two maxima are at the limit of discrimination when their angular distance is such that the maximum of one line falls on the minimum of the other. If the angular distance is smaller, the two lines merge into one - they are indistinguishable. If we apply this criterion, it turns out that the resolving power of the diffraction grating is proportional to the total number of slits and the order of the spectrum

$$R = Nm. \tag{31.8}$$

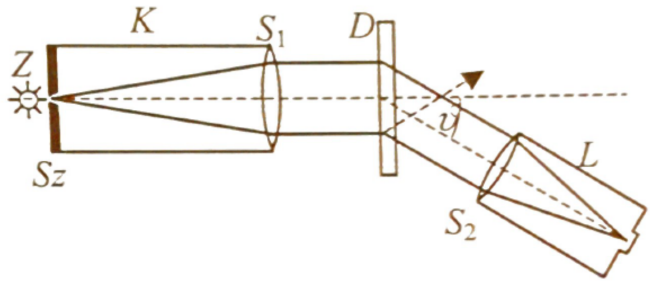


Figure 31.4. Spectrometric table; D - diffraction grating, K - collimator, L - scope, S_1, S_2 - lenses, Z - light source, S_z - slit

Principle of measurement

The constant of the diffraction grating d is the distance between the centers of adjacent slits. In order to find this quantity, we will use the formula (31.5), which after the transformation will take the form

$$d = \frac{m\lambda}{\sin\vartheta} \tag{31.9}$$

The investigation of the diffraction grating are done using light of a specific wavelength, most often sodium light ($\lambda = 589.6$ nm). The value of ϑ angles for individual rows is read using a spectrometer equipped with an exact angular scale. The ray traces in the grating spectrometer are shown in Fig. 31.4. The divergent light from the Z lamp enters the K *collimator* consisting of the aperture S_z and the lens S_1. Since the slit is at the focal point of the lens, it is parallel from the collimator. After passing through the D diffraction grating, the beam is focused by the S_2 lens of the L scope, thanks to which we observe a sharp image of the slit. The scope is equipped with a cross made of spider threads, which allows for precise setting of the slit image in the field of view.

The collimator is firmly attached to the spectrometer base, while the telescope is attached to the protractor and can be rotated about the spectrometer axis. The angular position of the telescope can be read with high accuracy using the angular scale provided with a vernier.

Measurements:

1. Using the telescope, look for the image of the gap without a diffraction grating. In the absence of an image, adjust the gap width, position and focus of the telescope.
2. After setting the gap image at the intersection of spider threads, write down the position of the telescope. It is equal to the position of the zero band ϑ_0.
3. Place the diffraction grating of interest one after the other on the spectrometer table.
4. List the positions of the higher order fringes located on the left (ϑ_l) and right (ϑ_r) of the zero line.

Report:

1. Determine deflection angles for each order for the tested grating.

2. Determine the diffraction grating constant for each measurement $d = \dfrac{m\lambda}{\sin\vartheta}$, where m - is the fringe order, $\vartheta = |\vartheta_0 - \vartheta_l| = |\vartheta_r - \vartheta_0|$ - the angle of light deflection.
3. Determine the constant of each diffraction grating as the average value. Determine their uncertainty (σ_s - the standard deviation of the arithmetic mean).
4. Present the final results of the experiment (properly rounded).
5. Write down the final conclusions

Keywords:

- wave nature of light, Huyghens principle,
- interference, amplification and weakening conditions expressed by phase and by way,
- coherence, diffraction on a single aperture: image dependence on the aperture width, location of minima, maximum width,
- two apertures: maximum condition, from which the distance between the fringes depends, and what is the relative intensity?
- Diffraction grating: construction, width of main maxims, resolution,
- spectrometer structure, vernier

32. Optical emission spectra study

Exercise goals:

- Identifying elements based on their spectrum
- Examining spectra of selected sources of white light

Introduction

Spectrum is a very broad term in science and technology. In broad meaning it's a dependence of signal intensity from its frequency. Spectrum may concern electromagnetic waves (microwaves, light, x-rays), acoustic waves (infrasound, sound, ultrasound) and other signals. Branch of science concerned with examining spectra is *spectroscopy.* Spectroscopy gives us a lot of information about different phenomena and properties of matter. Because spectroscopy is a very broad branch of science in this exercise we will focus on small segment of examining optic spectra. *Optic spectrum* is a dependence of luminescence from frequency or wavelength.

Methods of obtaining optic spectra

Light commonly describes electromagnetic waves visible to human eye (wavelengths between 380 and 780 nm). In engineering light is a broader term: it describes electromagnetic waves which behave according to laws of optical geometry. Besides visible light it also applies to close infrared and close ultraviolet.

To observe and register spectra in visible range we use *spectrometers* equipped with elements that splits light (prisms or diffraction gratings). In modern spectrometers split light falls on light sensitive charge-coupled device, and then is registered on a computer. Figure 32.1 shows schematic spectrometers equipped with prism and reflective diffraction grating. In reality spectrometers are more complicated, and more advanced devices are equipped with several elements that split light.

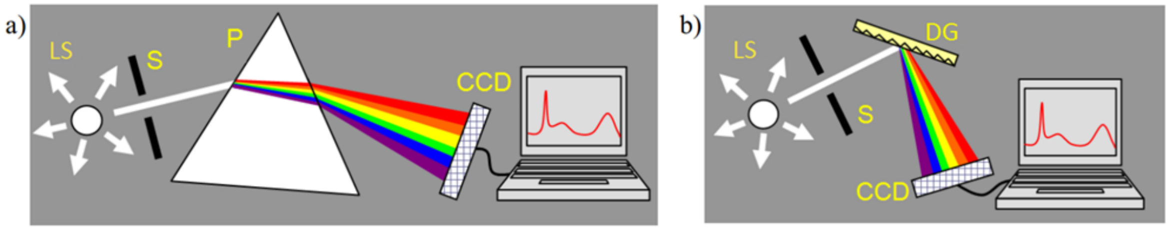


Figure 32.1. Schematic model of optical spectrometer a) equipped with prism b) equipped with reflective diffraction grating. Markings: LS - light source, S - slit, P - prism, CCD - charge-coupled, DG - reflective diffraction grating.

Types of optic spectra

There are a lot of methods of classifying spectra. Two basic ones are shown here.

1. Based on mechanism of generation spectra can be divided to:
   a) *emission spectra* – received as a result of medium radiating light (Figure 32.2a)
   b) *absorption spectra* – received after white light travels through examined medium (Figure 32.2b)
2. Based on character (representation) of spectrum (Figure 32.3)
   a) *linear* – consisting of series of thin lines corresponding to specific wavelengths; monoatomic gases and metal vapours are sources of this spectra
   b) *band* – consists of large number of lines close to each other, creating quite wide bands as a result: diatomic gases or particles are sources
   c) *continuous* – consists of waves of all lengths: solids and liquids are sources.

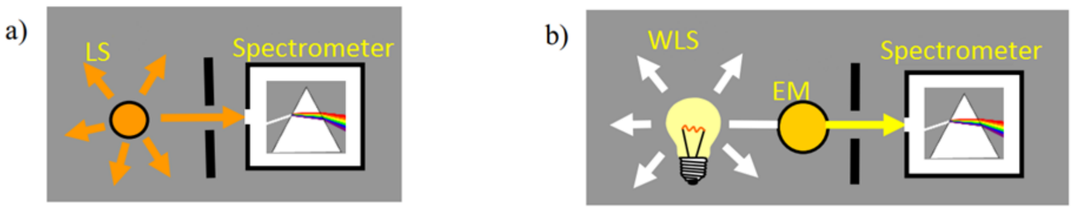


Figure 32.2. Methods of generating a a) emission, b) absorption spectra. Markings: LS - light source, WLS - white light source, EM - examined medium.

Why does matter shine?

To answer this question we must examine structure of atom. Atom consists of small, heavy, positively charged nucleus and orbiting around it negatively charged, small electrons. Movement of those electrons takes place on so called stationary orbits, and

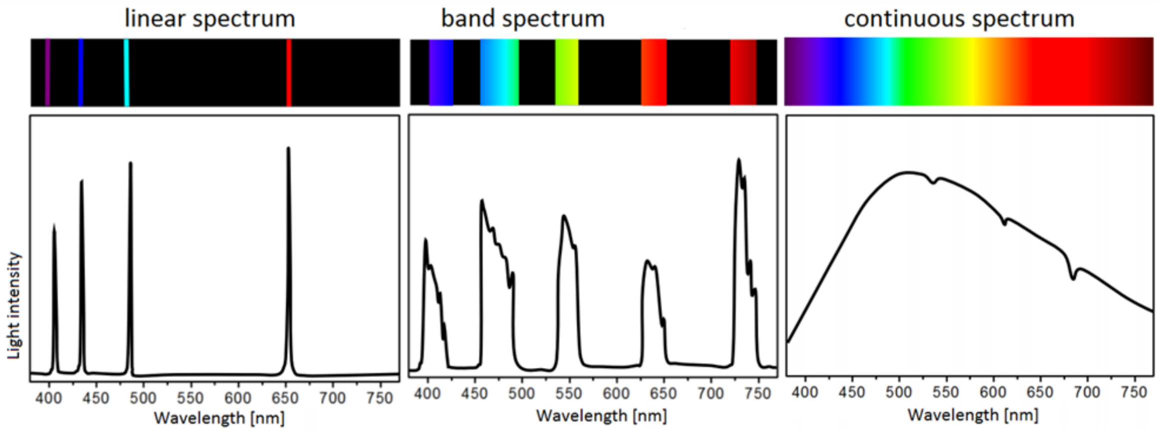


Figure 32.3. Examples of emission spectra: linear, band, continuous. Top images shows separated rays of light, bottom registered spectra

the energy of electron on each orbital is specified. In other words: energy of electron in atom is quantified. When electron moves from higher energy orbital (orbital further from nucleus) to lower energy orbital electron emits quantum of energy equals to:

$$E = h\nu = E_n - E_m, \tag{32.1}$$

where h - Planck constant, ν – frequency of emitted electromagnetic wave, E_n and E_m – energies of electron on nth and mth orbital. Using dependence between frequency and wavelength $\nu = c/\lambda$ we can transform formula (32.1) to:

$$\lambda = \frac{hc}{E_n - E_m}, \tag{32.2}$$

where λ – length of emitted electromagnetic wave, c – speed of light in vacuum.

This formula shows that if energies of electrons in atom can have only certain values, it can only emit waves of certain wavelengths. Based on analysis of spectrum we can learn a lot about structure of an atom, or on the other hand identify type of element that emits waves. Waves emitted by atom in a way described above fall in range of infrared, visible, ultraviolet of X-rays. If waves are in range of visible light we say that object shines.

So far we talked about generating linear spectra. Why are spectra of complex gases, liquids and solids different from spectra of singular atoms shining? It's because system consisting of multiple atoms increases the number of possible energy levels. For example: hydrogen molecule H_2 consists of two electrons and two protons, but it's spectrum is much more complex than spectrum of singular hydrogen atom. Internal energy of molecule additionally consists of energy of molecular vibrations and rotations. As a result H_2 molecule has numerous spectral lines compared to only four in visible range for singular atom H. The more complex molecule and atoms building it are, the more complex its spectrum is. For liquids and solids number of lines is so large that we observe continuous spectrum.

Examples of applying optical spectroscopy:

1) To control the quality of plates in rolling mill, electric arc is crated, which makes shining atoms to evaporate from material. by examining linear spectrum of this vapour, we can determine composition of material (using characteristic lengths of lines) and proportions of atoms (based on ratio of intensity of lines of specific elements)

2) Based on spectrum emitted by a star astronomers can determine its composition and velocity at which it's moving relative to Earth. Using subtle changes in star spectra it was concluded that the universe is expanding.

What is a white light?

White light is commonly defined as *combination of all colours* (all waves in visible range). If we observe white sunlight spliting in raindrops we see multicolour rainbow. However for human eye we can create impression of white light in multiple ways. It's a result of the fact that photoreceptor cells (cones) in human eye are sensitive to three basic colours: red, green and blue. Mixing them in different proportions we can create impression of white light or create new colours. Sometime white light from different sources falling on white piece of paper looks almost identical, but have significantly different spectra. Also colourful painting will look different when it's lit by, for example, light bulb, compact fluorescent bulb, or LED lamp, despite the fact that theoretically all of the emit white light. It's caused by different methods of creating light in those sources and therefore different spectra.

Let's examine ways in which we can create white light. As it was mentioned in order to make an atom shine its electron must obtain energy necessary to "jump" to higher orbital to later lose this energy by emitting electromagnetic wave. This energy can be delivered in different ways. Easiest way is heating up to high temperatures. For example surface of the Sun, heated to 5800 K emits intense white-blue light. Phenomena of light being created as a result of high temperatures is used in *classic light bulbs*. Its tungsten filament is heated to 2700 K and shines white-yellow light. However this method is inefficient. Only about 3-5% of light is in visible range, rest is invisible to human eye infrared radiation. *Compact fluorescent bulb* is a more efficient source of white light. In this bulb vapours of mercury are stimulated to shine by electric discharges. Light emitted by mercury falls on phosphor which shines thanks to fluorescence. Spectrum of this lamp is considerably different than sunlight and because of that colours of for example a painting looks different than in sunlight. Other source of white light is *LED lamp*. Usually it's a collection of light emitting diodes covered in phosphor and placed in a casing intended for light bulbs. Diodes, thanks to effect of electroluminescence emit blue light which causes phosphor to shine. Yellow-green-red light emitted by phosphor is mixed with blue light of diode which gives white light. If spectrum of white light contains a lot of blue light it's called *cool white light*, if there isn't much blue light we call it *warm white light*.

Measuring system

Measuring system (Figure 32.4) consists of spectrometer (range between 300 – 1000 nm) connected to computer. Using optic fibre spectrometer can receive light from seven different sources marked A, B, C and 1, 2, 3, 4. Spectrometer is operated using *Overture* program, which should be launched after turning the computer on. Symbols of selected functions are on station.

Exercise consists of two stages. First stage is identifying elements in spectral tubes. Lamps on station contain: A – mixture of monoatomic gas and metal vapour; B – monoatomic gas; C - simple diatomic gas. During identification small resolution of spectrometer has to be taken into account. Because of that if distance between lines is smaller than 3 nm, they can blend together. Second stage is observing white light coming from different sources: 1 - compact fluorescent bulb, 2 - LED lamp, 3 - LED RGB lamp, 4 - light bulb.

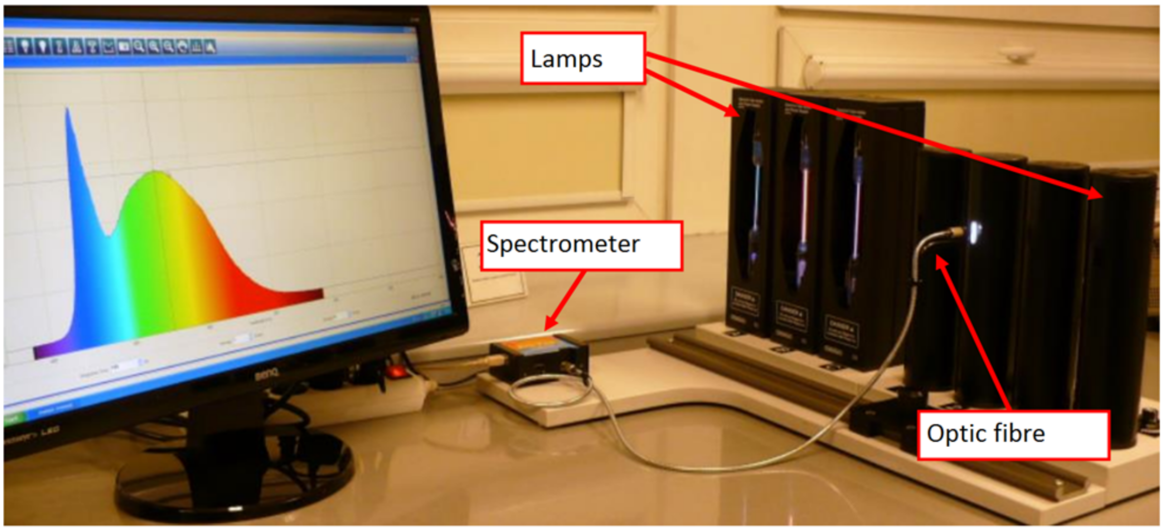


Figure 32.4. Experimental system used to examine optic emission spectra

Course of exercise

A. Identifying gases in spectral tubes

1. Turn on lamp A and move the end of optic fibre in front of it. Regulate position of fibre and "measurement integration time" so spectrum fits on the screen. Click the "colour" icon to show real colours of spectrum.
2. Using computer mouse move the cursor to the maximum of each line. Write down its wavelength and intensity. If a line is wider note that it can be multiple spectral lines close together.
3. Repeat measurements for lamps B and C.
4. Compare obtained spectral lines to spectra in "Spectral tables" added to this exercise (see table C.10) or to spectra in *Spektruś* program.

5. Specify elements in lamps A, B and C and note down findings

B. Observing spectra of selected light sources of white light

1. Turn on lamp 1 (compact fluorescent bulb) and register spectrum.
2. Write down positions of bands and their intensity and compare them to lines emitted by mercury. Write down the conclusions.
3. Turn on lamp 2 (LED lamp) Write down maximums of spectrum. Write down the conclusions.
4. Turn on lamp 3 (LED RGB lamp) and using pilot set white light (point the pilot to the hole in the back of the cover). Note down wavelengths for which you observe maximum values of spectral bands.
5. Using pilot change colour to for example yellow, purple, orange. Analyze spectra and write down findings.
6. Turn on lamp 4 (light bulb). Using potentiometer set maximum voltage on bulb, corresponding to maximum temperature of filament. Register spectrum using "shutter' icon.
7. In the same way register 3-4 more spectra while lowering voltage.
8. Compare obtained spectra for different temperatures of filament. Notice minimal values of wavelengths emitted by bulb for different temperatures of filament.
9. Write down the final conclusions

Keywords:

- linear, band and continuous spectra
- element identification based on spectra
- spectrometer construction
- absorption spectra, photoluminescence, Stokes rule

33. Determination of the refractive index of a liquid using an Abbe refractometer

Introduction

When a light ray runs from an optically thinner to optically denser medium, e.g. from air to glass, it is refracted at the interface of the media, creating a smaller angle with the normal to the surface (*refraction angle*) in a denser medium than in a thinner medium (the corresponding angle is called the *angle rainfall*). In the case of the reverse rays of the rays, the angle of incidence is smaller than the refractive angle. Each angle of incidence α corresponds to a different angle of refraction β, but the ratio of the sines of both angles has a constant value for a given pair of media and for a given wavelength of light. It is quantified by the following equation:

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}. \tag{33.1}$$

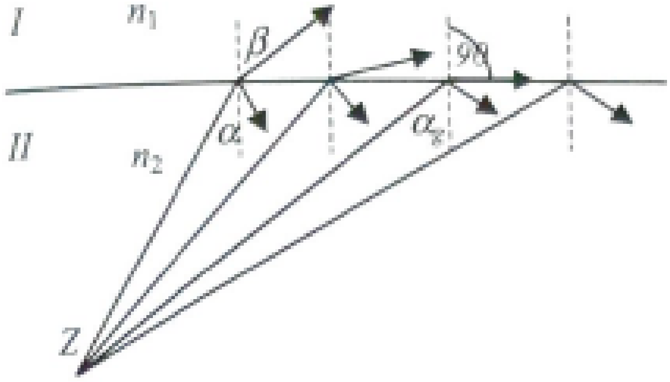


Figure 33.1. Illustration of complete internal reflection

The above formula expresses *the law of refraction* (*Snell's law*), and the values n_1 and n_2 are called *the absolute refractive indices* of medium 1 and medium 2. The absolute refractive index is also determined by the ratio of the speed of light in a vacuum c to the speed of light in a given medium v:

$$n = \frac{c}{v}. \tag{33.2}$$

Since the speed of light is greatest in a vacuum, the absolute refractive index is greater than one for material media.

The refraction of light at the border of two material media is determined by their *relative refractive index*:

$$n_{21} = \frac{n_2}{n_1}. \tag{33.3}$$

In real conditions, light refraction often occurs at the interface between air and a liquid or a solid. In this situation it can be assumed that air has a refractive index very close to the value for vacuum ($n = 1$) and that equation (33.1) determines the absolute refractive index of the liquid or gas.

Total internal reflection

Snell's formula takes a particularly convenient form for this internal reflection occurring at a limit angle or greater. The essence of the phenomenon of total internal reflection is shown in Fig. 33.1.

The rays from the optically denser medium II to the thinner one deviate from the perpendicular the more the greater the angle of incidence α. The refraction angle reaches the value of 90 ° at a certain angle α_g, called the limit angle - the ray does not go to the medium I. So we can see that at the limit angle and higher incidence angles the rays cannot pass to the rarer medium - they are completely reflected.

For the angle of incidence equal to the limit angle, the refraction law takes the form:

$$\frac{n_1}{n_2} = \frac{\sin \alpha_g}{\sin 90^\circ}. \tag{33.4}$$

If we know the refractive index of the denser medium and measure the limit angle α_g, we can determine the refractive index n_1 of the rarer medium. We measure the limit angle with a refractometer.

For rays going from a thinner to a denser medium, complete reflection does not occur.

Construction of an Abbe refractometer

The main part of the Abbe refractometer are two rectangular prisms - P_1 i P_2 (Fig. 33.1) made of flint glass with a high refractive index. The prism P_1 can be deflected by rotating it around axis O. After pivoting, a few drops of the tested liquid are placed on the hypotenuse surface of the prism P_2, which after pressing the prisms forms a thin, plane-parallel layer.

A divergent beam of light from any source falls on the P_1 prism, and then it walks at different angles of incidence to the hypotenuse, encountering the medium with a lower refractive index, which is the tested liquid between the prisms. The boundary between the P_1 prism and the liquid is the surface on which total internal reflection can occur - for some angles of incidence, it will not.

Suppose that a ray of light 2 in Fig. 33.2 falls at an angle slightly smaller than the limiting angle; then rays above ray 2 will be completely internally reflected, while rays between 2 and 3 will pass to the second prism P_2 and then to the finder L_1. The rays that fall on the prism P_1, in other places, create a beam coming out with P_2 contained entirely between the marked rays 2 and 3.

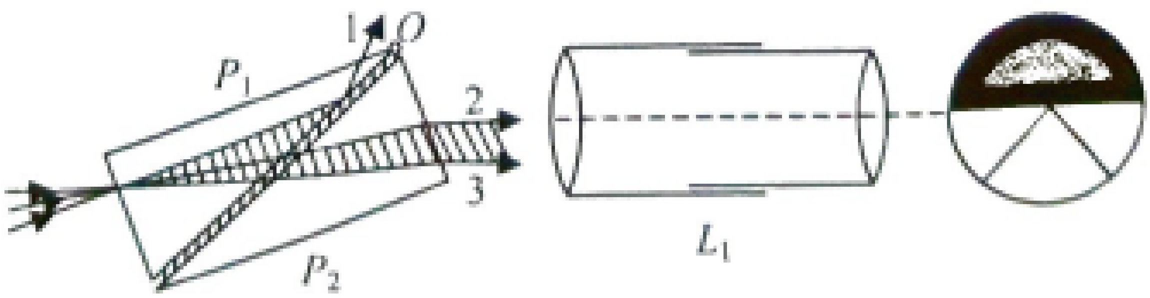


Figure 33.2. Measurement principle with an Abbe refractometer; L - scope, O - axis of rotation of the moving prism P_1, P_2 - stationary prism

With the described beam path, in the field of view of the telescope we can see a bright area (from beam 2 downwards) and a dark area (above beam 2). The position of the border between light and dark areas depends on the value of the refractive index of the liquid. This border is guided to the center of the field of view of the stationary telescope by the rotation of the P_1 P_2 prisms.

The universal Abbe refractometer, which we use in the exercise, is a more developed structure (see fig. 33.3), thanks to which it allows the use of white light and direct reading of the refractive index value.

The rotation of the P_1 and P_2 prisms is coupled to the movement of the K scale prepared on the basis of equation (33.4) in such a way that the refractive index is directly read from it. We make the reading with the L_2 scope, through which we can see two divisions: one showing the value of the refractive index and the other giving the percentage of sugar in the aqueous solution. The latter is only used to determine the sugar content. In some constructions, the refractometer has only one telescope - we can see both the image and the scale through it.

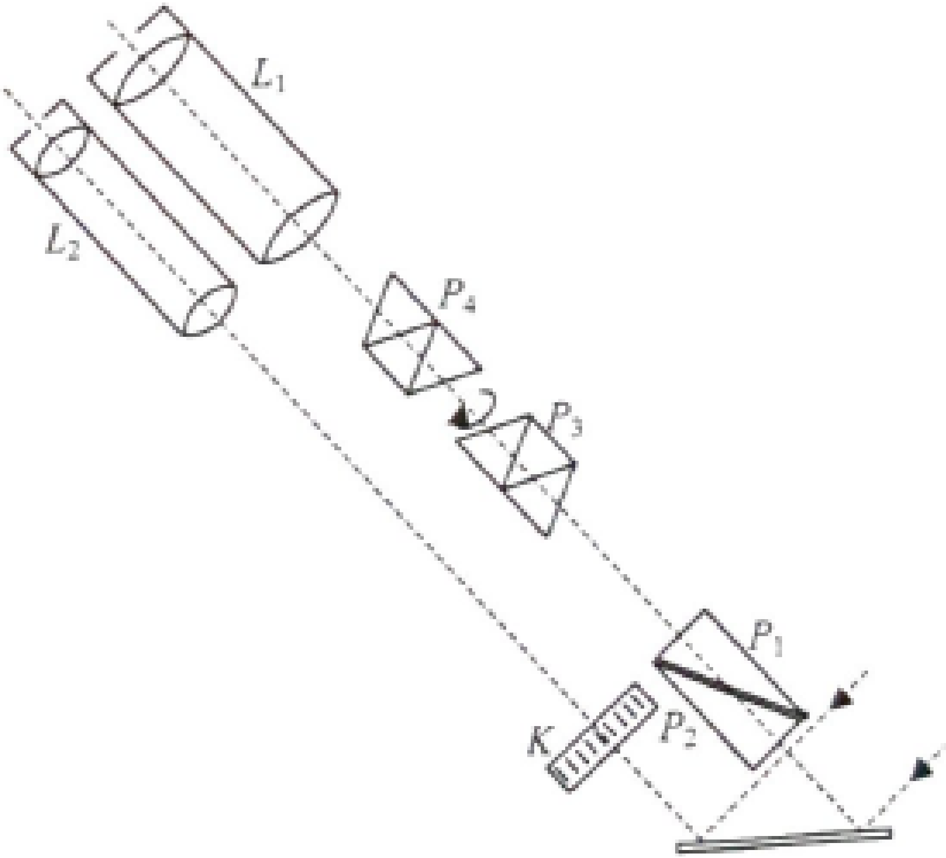


Figure 33.3. Construction of an Abbe refractometer

In the prisms of the refractometer, as well as in the tested liquid, the phenomenon of dispersion occurs, as a result of which white light is split. As a result, the line between the field of view is not sharp, but rather colored and blurred.

In an Abbe refractometer, the fission is compensated by a system of two prisms P_3 and P_4. Each of them consists of several, usually three, single prisms made of different types of glass, with different refractive indexes. The breaking angles of these prisms are selected so that the light of the yellow sodium line is not subject to any deviation. Red and purple rays are deflected in opposite directions. The prism gives the spectrum as seen straight ahead and is therefore called em á vision direct. The light diffusion caused by two identical prisms depends on their mutual orientation. When they are arranged in parallel, the refractions caused by each of them add up, while when one of them is rotated 180° around the optical axis, the fission is canceled - the light behind the other prism is white. For the other P_3 and P_4 prism settings we have intermediate cases. In the refractometer, this system is used to compensate for the dispersion caused by the tested liquid.

Measurements:

1. Turn on the table lamp and use the mirrors to illuminate the field of view of both spotting scopes.
2. Set the sharpness of the spider's threads and the scale in the telescopes.
3. Fold back the prism P_1, check that its surface is dry and clean, then apply a few drops of the test solution to it with a pipette and press the prism.
4. Use the P_1, P_2 prism dial to align the border to the center of sight of the telescope light and dark field. Compensate for color split.
5. Read the value of the refractive index of the liquid in the L_1 scope.
6. Repeat the measurements for solutions with subsequent concentrations (also for the concentration designated "X"). Remember to clean and dry the prism surfaces before using the new solution.
7. For pure glycerin (100% concentration) measure the refractive index as a function of temperature (T). An ultrathermostat (see 7) combined with a refractometer is used to regulate the temperature.

Report:

1. Plot the dependence of the refractive index on the concentration of the solution $n = f(C)$.
2. Plot the relationship $n = f(T)$ the refractive index on the temperature for 100% glycerin.
3. Draw the appropriate error rectangles on both graphs.
4. Using the experimentally obtained dependence of the refractive index of the solution on the concentration of glycerin and the determined refractive index of the unknown solution "X" determine its concentration.
5. Determine the accuracy of the concentration determined.
6. Write down the final conclusions

Keywords:

- angle of incidence, refraction and reflection, refractive law, absolute and relative fracture rates
- internal reflection
- construction of an Abbe refractometer, rays running through the system
- dispersion, dispersion compensation

34. Investigation of the polarization plane torsion caused by solutions using a polarimeter

Introduction

The light from natural sources is *non-polarized*, i.e. the vibrations of the light vector take place perpendicular to the direction of the rays, but in all possible planes on which this direction lies (Fig. 34.1 - in front of the polarizer). When we place a *polarizer* in the path of a beam of non-polarized light, it will only pass through those rays in which the vibrations take place in one plane. After passing through the polarizer, the light is *linearly polarized* - the ends of the light vectors lie on a straight line. Parallel lines on the polarizer, which are not actually visible, show the characteristic directions of the plate's polarization.

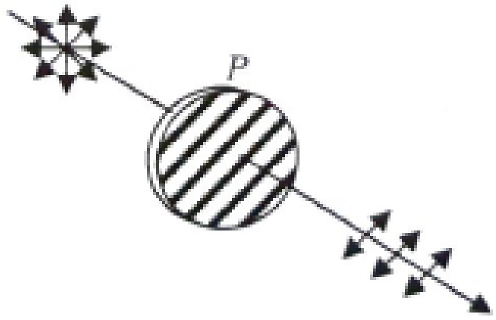


Figure 34.1. Production of linearly polarized light; P - polarizer

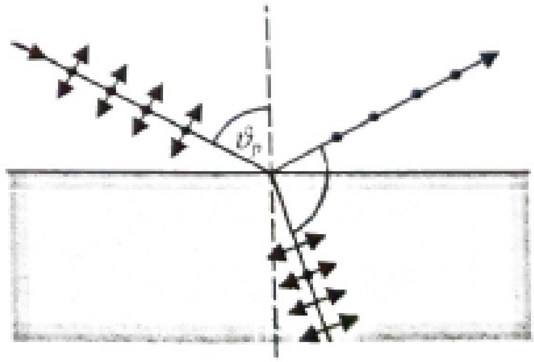


Figure 34.2. Polarization on reflection; φ_p - Brewster angle

Polarization by reflection

As a result of light reflection from the boundary of two media (Fig. 34.2), both the reflected and refracted rays become partially polarized. The degree of polarization depends on the angle of incidence - if we choose it so that the angle between the reflected and refracted rays is straight (Fig. 34.2), the reflected ray is completely polarized in the plane perpendicular to the plane of incidence, while the refracted ray is partially polarized, with the predominance of vibrations in a plane parallel to the plane of incidence. The φ_p angle is called *the angle of complete polarization* or *the Brewster angle*. The degree of polarization of the refracted beam can be increased by passing it through a set of parallel plates.

Polarization in anisotropic crystals

If the crystal properties depend on the direction, the crystal is *anisotropic*, otherwise it is *isotropic*. In the phenomenon of double refraction by anisotropic crystals, the incident beam is split into two: ordinary (o) and extraordinary (e), with perpendicular vibration planes. The extraordinary beam does not obey the law of refraction (Snell's law). The direct cause of the different behavior of o and e rays is their generally different speed. The speed of the ordinary ray v_0 is constant in all directions of the crystal, while the speed of the extraordinary ray varies, depending on the direction, from the value of v_0 to the value of v_e, where v_e for some crystals is less than v_0 for others - greater. The direction in an anisotropic crystal for which $v_e = v_0$, is called *the optical axis* of the crystal. The ray that runs in the crystal parallel to the optical axis is not refracted twice. If we remove one of the beams, we get linearly polarized light at the output of the anisotropic crystal. A common device that uses the described phenomenon is the Nicola prism.

Polarization and dichroism

Some double refractive crystals have a property called *dichroism* in which one of the polarization components is absorbed in the crystal much more strongly than the other, which passes with a slight attenuation. This property is the foundation upon which the widely used *polaroids* function is based. Instead of homogeneous crystals, it is possible to use a large number of small crystals arranged in a plastic plate so that their optical axes are parallel.

When we place two polarizing plates on the axis of the running light beam, one of them will function as a polarizer, and the other - as an analyzer. By turning the analyzer, we find that in some positions the system almost does not pass any light, and in positions that differ from those by 90° the light intensity is maximum. This is of course related to the angle that the polarization directions form with each other in both polaroids. The intensity of the light coming from the analyzer as a function of the mentioned angle is described by the Malus law:

$$I = I_m \cos^2 \varphi, \tag{34.1}$$

where I_m corresponds to the angle $\varphi = 0$.

Reducing light vibrations to one plane, i.e. linear polarization, is not the only way to arrange light vibrations. There may also be a light where the end of the light vector traces a helical line around the propagation direction. We are talking then about circular or elliptical polarization.

Circularly polarized light is created as a result of the superimposition of two coherent linearly polarized waves in mutually perpendicular directions (they can be ordinary and extraordinary rays) with a phase difference of 90° and equal amplitudes. The resulting vibration will be circular according to the principle of adding perpendicular vibrations (see Lissajous figures). When the amplitudes of the component vibrations are different, the polarization is elliptical.

If linearly polarized light passes through some substances, called optically active substances, the plane of polarization is twisted. Optically active substances exist in two forms, which exhibit the same torsion capacity, but in opposite directions. Hence, optically active substances are divided into right- and left-handed. The particles of right- and left-handed substances differ in structure, as well as the image and the object in a plane mirror, they are the so-called enantiomorphs. In optically active substances there is the so-called an asymmetric carbon in which each of the valences is saturated with a different atom or group of atoms.

An example of such a substance can be tartaric acid, the two enantiomorphic forms of which are presented in Fig. 34.3. Strong left twisted (left) and right twisted properties Polarization planes also show torsional (right) sugar solutions.

Measurements and calculations

The twist angle of the plane of polarization through the solution with concentration c is determined by *the Biot formula*:

$$\alpha = [\alpha] \cdot l \cdot c, \qquad (34.2)$$

where: $[\alpha]$ - specific torsional ability, l - light path length in the solution.

To measure the twist angle, first place a clean solvent in the path of the light beam and read the position of the analyzer α_o and then replace the solvent with the test solution and read the position of the analyzer again - α_c. The angle of twist I am looking for is $\alpha_c - \alpha_0$. Devices that are used to study the rotation of the plane of polarization are *polarimeters*. Polarimeters for measuring sugar concentration are called *saccharimeters*. The main elements of the polarimeter (Fig. 34.4) are: the P polarizer and A analyzer (Nicol's prisms) and the R tube between them containing the optically active substance solution.

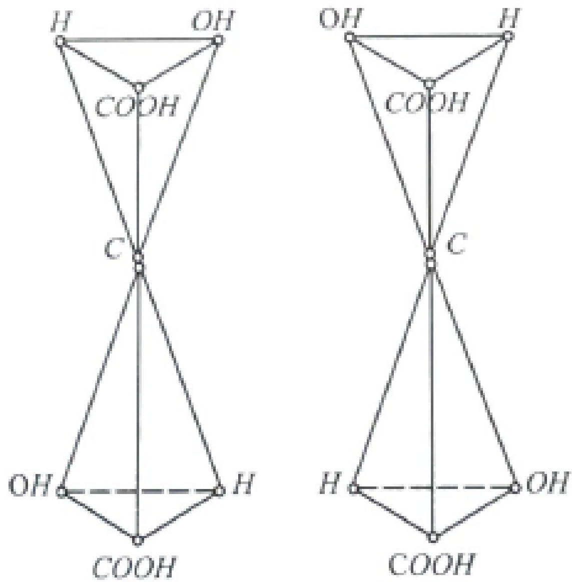


Figure 34.3. Structure of a molecule of left-handed tartaric acid (left) and right-handed (right)

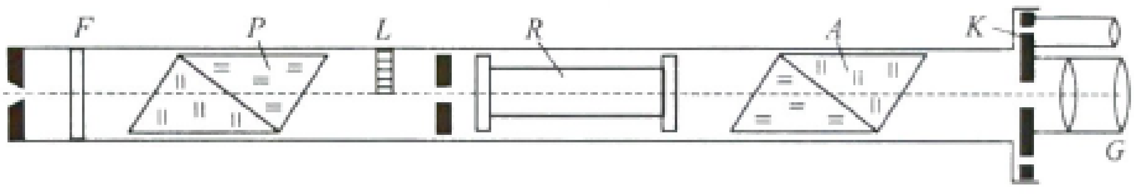


Figure 34.4. Construction of a Laurent polarimeter; F - light filter, P- polarizer, L - instrument halftone, R - tested liquid in the tube, A - analyzer, K - protractor, G - scope

The light enters the instrument through a filter, giving off a narrow wavelength range. The analyzer is connected to a protractor. The scope is used to observe the light passing through the system, and the magnifier is used to accurately read the angles of rotation of the analyzer.

The essence of the measurement comes down to the most accurate determination of the position of the analyzer at which the illumination of the field of view is constant. When observing with the naked eye, we set the analyzer to the maximum extinction of the light - in these conditions the eye is most sensitive to changes in brightness. In factory polarimeters, *the semi-shade device* helps to precisely position the analyzer. It is a plate that slightly twists the plane of polarization, but only for part of the beam. This is the reason why the field of view of the telescope is divided into fields, usually differently illuminated. Depending on the design, the fields may have the shape of two semicircles or a wide central strip and two side fields. By turning the analyzer, we can bring the illumination of the entire field to a small but even position. Minimal analyzer rotation causes lightening of one of the fields. In this situation, the human eye can distinguish with high accuracy (about 0.1°) whether the fields are illuminated in the same way or whether their illumination differs. This is the most sensitive position of the analyzer. In other positions, we can obtain high contrast fields or the entire bright field.

In order to find the correct torsional capacity, we measure the torsional angles for different bracings and use the fact that the relationship between these values, given by equation (34.2), is linear. The slope a_{reg} is computed by linear regression. The same coefficient appears for c in the mentioned equation as $([\alpha]\cdot l)$. By comparing both values, we obtain the desired value of the proper torsional ability

$$[\alpha] = \frac{a_{reg}}{l}. \tag{34.3}$$

Measurements:

The measurement uses sugar-in-water solutions placed in tubes with a length of $l = (0.185 \pm 0.005)$ m.

1. Adjust the telescope in such a way as to obtain the visual acuity of the dividing line observed in the polarimeter of a field with different shading. Set the reading magnifier to the scale view sharpness.
2. Insert a tube of clean water into the inside of the polarimeter. Turn the analyzer to get an even (dark) field of view. Record the value indicated by the protractor. Repeat the measurement at least six times.
3. After removing the tube with clean water, repeat the measurements for the next tubes containing solutions of different concentrations c. Note the analyzer angle (α_A) and solution concentration (c).
4. Note also the angle observed for the sugar solution of unknown concentration in the tube marked 'X'.
5. Note the accuracy of the measuring device

Report:

1. Calculate the angles (α) of the torsion of the polarization plane by each of the solutions

2. Make a plot of $\alpha = f(c)$.
3. Using the linear regression method, determine the slope coefficient a_{reg} and its uncertainty Δa_{reg}.
4. Determine the proper torsional ability $[\alpha]$ for the aqueous sugar solution using equation $[\alpha] = \frac{a_{reg}}{l}$, where l - is length of the light path in solution. Calculate $[\alpha]$ uncertainty using the total or logarithmic differential method.
5. Find the sugar concentration in tube "X". Specify the measuring accuracy of this concentration.
6. Present the final results of the experiment (properly rounded).
7. Write down the final conclusions

Keywords:

- linear polarization, polarization by reflection and refraction, double refraction
- anisotropic and isotropic crystals, optical axis
- Nicola's prism, dichronism
- polarizer and analyzer, Malus law
- circle and elliptical polarization
- polarization plane torsion, asymmetric carbon
- polarimeter, half-shade device
- Biot's formula, vernier

35. Determination of the light efficiency of selected light sources

Exercise goals:

- determining illuminance as a function of distance from a source of light
- determining luminous efficiency of examined light sources (LED lamp, halogen bulb and traditional bulb)
- determining dependence of luminous efficiency from power consumption

Introduction

Patented by Thomas Edison in 1879 bulb is still very popular in home lighting. It's caused by low production cost, lack of stroboscope effect and electromagnetic spectrum similar to sunlight spectrum. Big disadvantage of light bulb is low luminous efficiency, only 2 – 4 % of energy consumed is converted to light. Because of that people were working on finding more economic substitutes for years. Currently a lot of different electrical light sources are available but their relatively higher luminous efficiency comes witch disadvantages. For over a decade fluorescent lamps can be found in our homes and efficient LED lamps in last few years. Big advantage of latter is long lifetime and high luminous efficiency (often more than ten times efficiency of traditional bulb). In this exercise we will focus on measuring latter parameter.

Determining the luminous efficiency of light source

The luminous efficiency η is defined as ratio of *total luminous flux* Φ_C emitted by a source to *power* consumed by source P

$$\eta = \frac{\Phi_C}{P}. \tag{35.1}$$

Consumed power can be calculated by multiplying *voltage* U by *electric current* I flowing through light source

$$P = UI. \tag{35.2}$$

Luminous flux Φ is the amount of energy going through surface in the unit of time. By measuring the energy of light waves going through surface surrounding light source we get total luminous flux of light source Φ_C. Measuring Φ_C is not easy because light sources have different shapes and energy of radiation they emit depends on direction (radiation is anisotropic). Special experimental methods are used to determine total luminous flux, such as photometric sphere or photometers mounted on special arm that allows measurement in different spacial configurations. Latter method allows the creation of spacial map of light distribution. In this exercise we will use simplified method of measuring total luminous flux based on the assumption that examined light source is isotropic point source. This means that the size is negligibly small and energy of emitted radiation is equal in all directions. This assumption is not true but later we will show that this simplification is justified.

Determining total luminous flux of isotropic point source based on measurement of illuminance.

Luminous intensity I_S is a basic photometric parameter. It's a ratio of luminous flux $d\Phi$ contained in infinitely small solid angle to value of this angle $d\omega$ (Figure 35.1)

$$I_S = \frac{d\Phi}{d\omega}. \tag{35.3}$$

The unit of luminous intensity is the *candela* (cd). This unit is strictly defined, it belongs to the SI system. Name comes from latin (candela – candle) and originally luminous intensity of 1 cd corresponded to luminous intensity of specially created candle. Today, obviously, this definition is not precise enough.

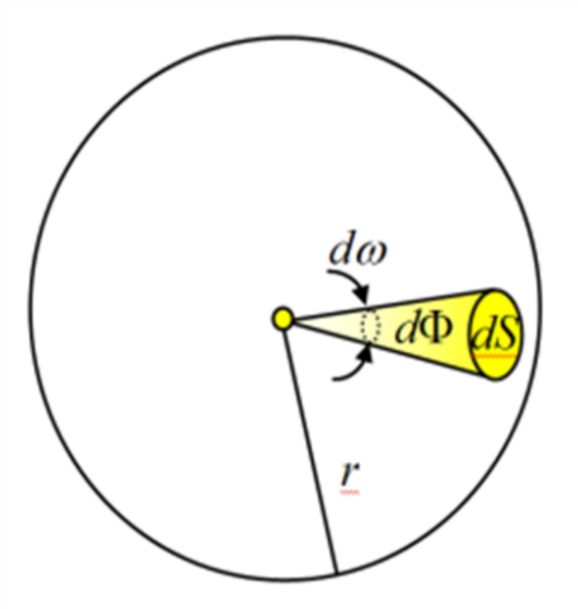


Figure 35.1. The radiation of the point light source

When we are dealing with isotropic source of light we can write luminous intensity as a ratio of total luminous flux to value of entire sphere $I_S = \Phi_C/4\pi$. After transformation we get the formula for total luminous flux

$$\Phi_C = 4\pi I_S. \tag{35.4}$$

The unit of luminous flux is the *lumen* (lm) defined as luminous flux of a light produced by isotropic point source that emits one candela of luminous intensity over a solid angle of one steradian (1 lm = 1 cd · 1 sr).

In our exercise we will determine Φ_C using different physical quantity – *illuminance* E - using lux meter. Illuminance is the ratio of luminous flux $d\Phi$ to *area* dS on which light falls

$$E = \frac{d\Phi}{dS}. \tag{35.5}$$

The unit of illuminance is lux (1 lx = 1 lm/m^2) Figure 1 shows point light source and a part of surface dS being illuminated by flux $d\Phi$. If as dS we use total surface of a sphere with radius r ($S = 4\pi r^2$), then $d\Phi$ will be equal to total luminous flux emitted by source Φ_C. Formula for illuminance will be

$$E = \frac{\Phi_C}{4\pi r^2}. \tag{35.6}$$

This equation shows that if we have isotropic point source measuring illuminance E from the distance of r from light source will allow us to calculate total luminous flux.

Can we treat bulb as point isotropic light source

Of course bulb is not a point source. It's proven, however that with a small approximation we can treat light sources as point sources if distance of measurement is at least 5 times bigger than size of light source. If our measurement will be made from far enough we can treat our light source as point source.

Isotropy is a more complicated problem. real sources more or less don't meet this criteria. for example so-called matt bulbs sends light evenly in all directions but in the direction of handle it's not emitted at all. Figure 35.2a shows so called candlepower-distribution solid showing intensity of radiation depending on direction from light bulb. Figure 35.2b shows illuminance depending on the angle (direction) from light bulb. It's easy to notice that we can observe highest values of illuminance for angles of 150° and 210°, for the 0° angle value of illuminance is equal to zero. Dotted line of figure 35.2b shows average value of illuminance. Average value is close to the value for 90°. We can approximate that the measurement of illuminance in the direction perpendicular to the axis of symmetry (90° angle) is equal to the average value of illuminance. This approximation can be used only for selected light sources and can't be used for professional measurements.

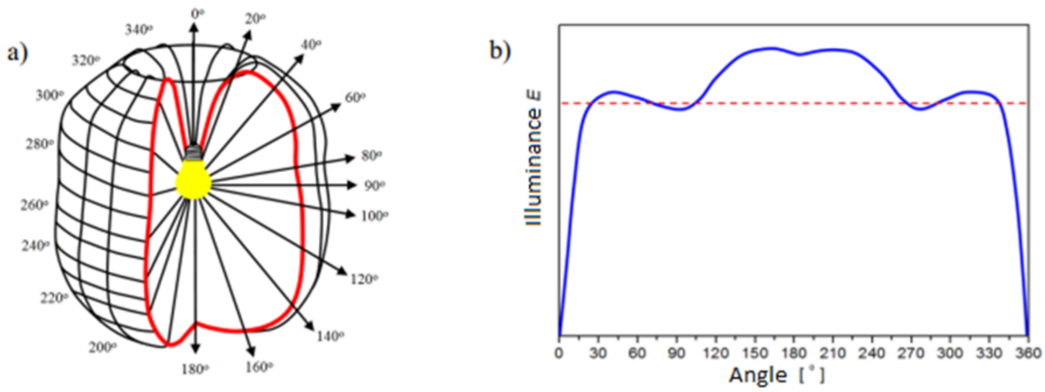


Figure 35.2. Example of light distribution by light bulb a) candlepower-distribution solid, b) illuminance as function of angle (direction) of emission. Dotted line shows average value.

Why is luminous efficiency of light bulb low?

Construction and working principle of light bulb hadn't change much in the last 100 years. It's a glass bulb usually filled with nitrogen, with tungsten wire inside (filament). As a result of flow of electric current filament is heated to about 2600 K. Every body at a temperature higher than 0 K emits electromagnetic waves. Most objects that surround us emit infrared light, invisible to human eye. Only after exceeding about 1000 K objects start so emit dark red light. Increasing the temperature causes emission of more colours: yellow, green, blue. We can observe combination of those colours as yellow-white colour. Still more than 95% of radiation is infrared. Luminous efficiency could be increased by increasing the temperature of the filament. For example if the temperature would be comparable to the temperature on the surface of the Sun (6000K) more than 40 % of radiation would be visible. Unfortunately this temperature is too high. Commonly used tungsten starts to rapidly evaporate in temperatures above 2600 K. In order to increase the temperature light bulbs started to be filled with halides (for example iodine, fluorine, bromine). Halides bond with atoms of evaporated tungsten, then, when they are close to heated up filament those compounds fall apart and tungsten is again settled on filament. This cycle is called halogen cycle and the bulb is called halogen bulb. Thanks to this cycle temperature of the filament in halogen bulb can be increased to 3000 K which gives us increase of luminous efficiency of about 30 % compared to traditional light bulb.

In past few years LED lamps are becoming more popular. LED lamp is usually a collection of light-emitting diodes covered in phosphor placed in casing designed for light bulbs. Working principles of LED lamp is completely different from light bulb, it's not heated up to high temperatures. Diodes emit blue light which excites phosphor to shine. Yellow-green light emitted by phosphor combined with blue light of diode gives white light. In case of LED lamp, unlike light bulb, all emitted light is visible. Losses are result of efficiency of the device, which means that luminous efficiency of LED lamps is much higher compared to light bulb.

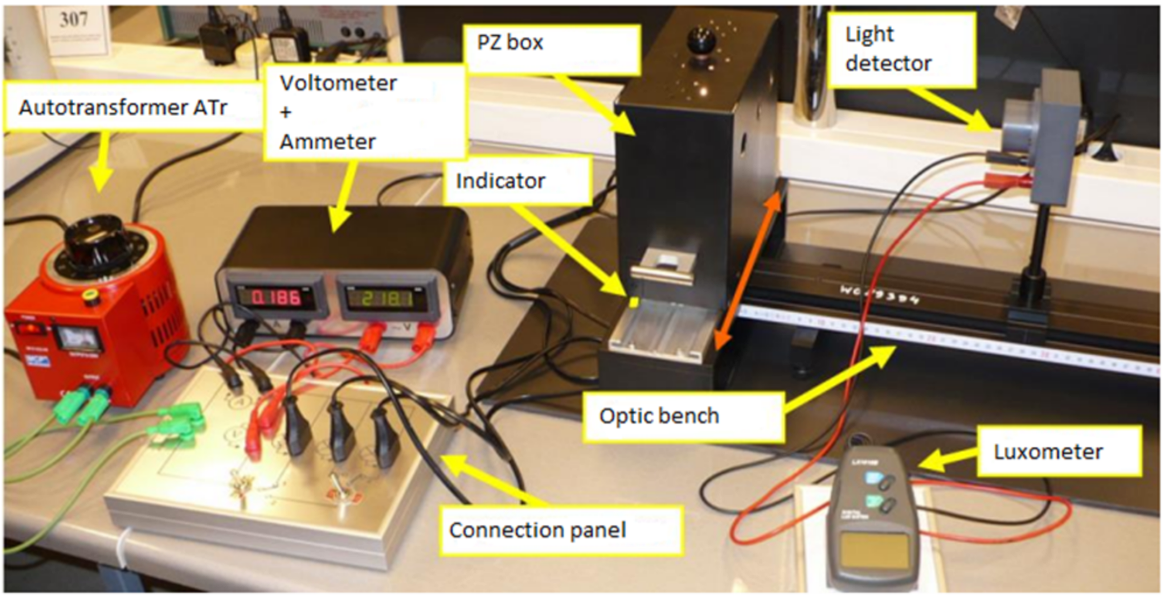


Figure 35.3. Experimental system used to measure luminous efficacy of selected light sources

Measuring system

Measuring system is designed to measure luminous efficiency of 3 light sources: LED lamp, halogen bulb and traditional light bulb (filled with nitrogen). Additionally it can measure luminous efficiency as a function of power consumption by sources. Figure 35.3 shows experimental system. Light sources are in box PZ which can be moved perpendicularly to optic bench. Light detector connected to lux meter in located on scaled bench which allows measurements of illuminance depending on the distance of light source. Autotransformer ATr is used to power the light sources. Light sources, voltmeter, ammeter and autotransformer are connected through connection panel. On this panel switches P_1 and P_2 are located, those switches are used to turn on selected light source. Fig 35.4 shows scheme of electric circuit.

Measuring luminous efficiency η of selected sources

In order to measure luminous efficiency of a light source box PZ should be moved in a way that light source is in front of light detector. Position "closest to yourself" means LED lamp is measured, middle position (box indicator on bench marker) means halogen bulb is measured, and position "farthest from yourself" is traditional light bulb. After turning selected source on values of voltage U and

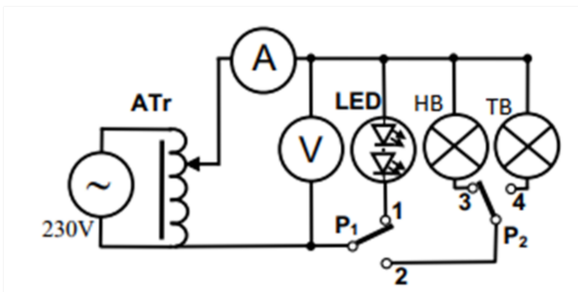


Figure 35.4. Electric scheme of power system and power measurement

current I should be measured. After that measurements of illuminance E depending on distance r should be made. In order to calculate total luminous flux Φ_C formula (35.6) should be used. Using this substitutions: $y = E$, $x = 1/r^2$ and $a = \Phi_C/4\pi$, we get $y = ax + b$ type formula. This is linear function where a is slope. Creating chart of illuminance as a function of reversed squared distance: $E = f(1/r^2)$ should give straight line. Applying the method of linear regression to this results can give the value of slope a, and after that total luminous flux $\Phi_C = 4\pi a$. Using measurement of voltage U and current I we can calculate power consumption $P = UI$. Finally luminous efficiency can be calculated using formula (35.1).

Measuring luminous efficiency as a function of power consumption

In order to calculate how luminous efficiency depend on consumed power light detector should be placed in constant distance r in front of bulb of interest. After that, perform measurements of illuminance E while changing Voltage using autotransformer. Using formulas (35.1), (35.2), and (35.6) we can create formula that will allow to calculate luminous efficiency of point isotropic source.

$$\eta = \frac{4\pi r^2 E}{UI}. \tag{35.7}$$

Power should be calculated using formula (35.2).

Course of exercise

A. Determining luminous efficiency η of light sources

1. Move PZ box to position “closest to yourself” so LED lamp is in front of lux meter detector.
2. Move switch P1 to position 1. Turn on voltmeter ammeter and autotransformer, next set voltage to 230 V. Write down values of voltage and current.
3. Make 10 to 12 measurements of illuminance depending on distance in range from 25 to 90 cm. Because dependence is not linear at first change distance by 2 cm, later by 5 cm, in the end by 10 to 15 cm.
4. Repeat measurements for halogen bulb (switches P_1 and P_2 in positions 2 and 3) and traditional bulb (switches P_1 and P_2 in positions 2 and 4)
5. Using those results plot on one chart dependences of illuminances from distances from light sources $E = f(1/r^2)$ for examined sources.
6. Using method of linear regression calculate slopes of generated lines a and their errors, afterwards total luminous flux $\Phi_C = 4\pi a$ and measurement errors.
7. Using received results and formulas (35.1) and (35.2) calculate luminous efficiency of examined sources and errors.
8. Compare results and write down findings.

B. Calculating luminous efficiency as a function of power consumption $\eta = f(P)$.

1. Set up traditional bulb 35 cm in front of lux meter detector. After that turn on bulb circuit and using autotransformer set voltage to 230V.

2. Make 10 measurements of illuminance while changing voltage by 10V. Each time note down values of voltage and current.
3. Using formulas (35.2) and (35.7) calculate each value of power and luminous efficiency.
4. Repeat measurements for halogen bulb.
5. On one chart plot values of luminous efficiency depending on consumed power $\eta = f(P)$.
6. Write down the final conclusions

Keywords:

- definitions of photometric quantities: luminous flux, luminous intensity (intensity), luminance, lighting,
- photometric units: candela, lumen, rivet, lux, light output,
- Lambert's law, photometer, light detector, lux meter.

36. Determination of the radius of curvature of the lens using Newton rings

Newton's rings

Circular interference rings, called *Newton's rings*, are created when a parallel beam of light strikes a system consisting of an exactly flat glass plate and a flat-convex lens lying on it (Fig. 36.1). Between the lens and the plate there is an air layer with a thickness d increasing with the distance from the axis of the system. The radius of curvature of the lens is several tens of centimeters and is much larger than the radii of Newton's rings; they are on the order of one millimeter.

The interference image results from the superimposition of rays reflected from the lower surface of the lens and the upper surface of the plate. Figure 36.1 shows the course of an example of a selected radius. Part of the beam incident vertically from above is reflected from the top surface of the plate (point B in the figure) and runs back upwards. The second part reflects off the inner surface of the (spherical) lens (point A in the figure) and also runs up to the microscope objective.

It should be noted that the radius of curvature of the lens in the drawing is much smaller than it actually is, to

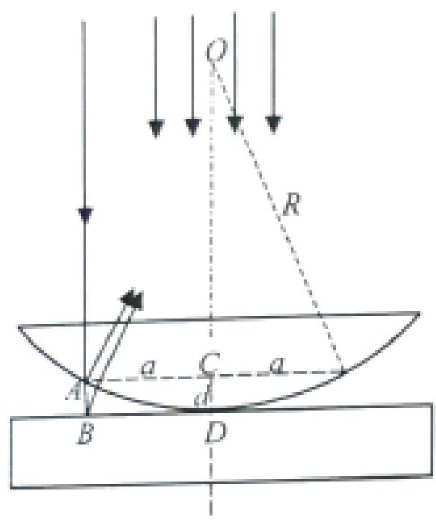


Figure 36.1. System for the production of Newton's rings

enable all details relevant to the phenomenon to be marked. In the drawing scale, the surface of the lens should be almost parallel to the surface of the plate, and the reflected rays - almost vertical.

The difference between the geometric paths of both radii is $2d$. The thickness of the slit d changes with the distance from the center point, so we can expect that for some thicknesses the amplification condition will be met and the ray incident there will be reflected as bright. Rays incident in other places will be dimmed after reflection. To calculate the optical paths, we assume that the refractive index of air is equal to unity, and also take into account the fact that reflection from a denser medium is accompanied by a phase change of 180°, which corresponds to an additional $\lambda/2$ road change. Given the above, we can write the condition for the formation of a bright interference ring:

$$2d + \frac{\lambda}{2} = m\lambda \quad (m = 1, 2, 3...), \tag{36.1}$$

where m is called the ring row; otherwise it is the number of the ring from the center. Based on Fig. 36.1, we can express the thickness of the air layer through the radius of the interference ring a:

$$d = R - \sqrt{R^2 - a^2} = R - R\sqrt{1 - \left(\frac{a}{R}\right)^2}, \tag{36.2}$$

remembering that $a/R << 1$, we can use series expansion of the square root expression and we get the form:

$$d = R - R\left[1 - \frac{1}{2}\left(\frac{a}{R}\right)^2 + ...\right] \approx \frac{a^2}{R}. \tag{36.3}$$

After combining the last equation with the equation (36.1) we get:

$$a = \sqrt{(m - \frac{1}{2})\lambda R} \quad (m = 1, 2, 3...). \tag{36.4}$$

The obtained equation determines the rays of bright interference fringes.

At the point of contact between the lens and the plate, a very thin air layer is formed, with a thickness many times smaller than the wavelength. The optical path difference arising between the rays at this point is due to the loss of only half the wavelength on reflection from the plate. As a result it is $\lambda/2$; in the middle of the interference image we observe a dark field. If the system is illuminated with white light, colored wide rings are formed which may overlap at the higher rows.

Measurements and calculations

In order to determine the radii of Newton's rings, we use a microscope adapted for this purpose. Place the plate with the lens in the microscope's field of view on a table that can be moved horizontally in two directions using micrometric screws. To enable simultaneous illumination of the system and observation of the image, on the optical axis of the microscope M (Fig. 36.2) we place a translucent plate P inclined at an angle of 45° to the direction of the rays. The plate reflects some of the rays from the source S and directs them to the system, where they are reflected and interfered with, and then pass through the plate P to the microscope objective.

The microscope eyepiece is equipped with a cross made of spider threads, thanks to which we can precisely set the selected fragment of the image in the field of view. By moving the table along the line through the center of the image in the X direction only, you can find the positions of the bright rings on the right of a_r, and on the left of a_l from the center. The radius of the fringe of the m order is the half of the diameter:

$$a_m = \frac{a_r - a_l}{2}. \quad (36.5)$$

If the relationship (36.4) is transformed into the form

$$a_m^2 = \lambda R(m - \frac{1}{2}), \quad (36.6)$$

it can be seen that it will be useful to plot in the coordinates $y = a^2$, $x = (m - 1/2)$, because the plot will then be a straight line. The slope coefficient is R. The value of this coefficient is obtained from linear regression; let us denote it a_{reg}, (not to be confused with the radius of the fringe!). The equation gives the final value of the radius of curvature of the lens:

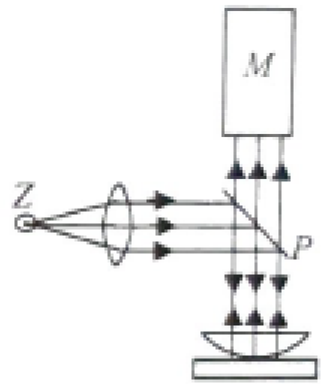


Figure 36.2. Newton's ring observation system: M - microscope, P - translucent plate.

$$R = \frac{a_{reg}}{\lambda}. \quad (36.7)$$

The lens's radius of curvature is determined by taking the wavelength $\lambda = 589.6$ nm.

Measurements:

1. Use the stage feed screws to measure the position of the next light rings to the right of the center. Do the same for the left edge of the rings. Take measurements for all measurable rings (it seems feasible to measure rings from m=3 to m=20).
2. Note the accuracy of the measuring device

Report:

1. Calculate the radii of Newton's rings (r [mm]) of subsequent rows (m).
2. Calculate the square of the radii of Newton's rings and the corresponding expression $(m - 1/2)$.
3. Plot $r^2 = f(m - 1/2)$.
4. Using the linear regression method, calculate the slope coefficient a_{reg} and its uncertainty Δa_{reg}. If some of the measurement points deviate from the straight line, these points should be thrown. The most likely cause of non-linearity is deformation of the lens near the point of contact with the plate, due to excessive pressure. The result is an enlargement of the radii of the low order rings.

5. Determine the radius of curvature of the lens $R = \frac{a_{reg}}{\lambda}$ (wavelength of light used in the system: $\lambda = 589.6$ nm, NOTE: $[a_{reg}]$=[mm^2] so change λ to [mm].).
6. Determine the measurement uncertainty of this radius (ΔR).
7. Present the final results of the experiment (properly rounded, unit [R]=[mm]).
8. Write down the final conclusions

Keywords:

- wave nature of light, electromagnetic waves, wavelength
- interference, blanking and amplification conditions, wave coherence
- phase change after reflection, optical path
- system for making Newton rings
- the condition for the formation of a bright ring, the band row and the difference of paths, the brightness of zero order

A. List of Exercises

MECHANICS

Ex.101 Determination of the speed of sound in the air by the phase shift method.
Ex.102 Determination of gravitational acceleration using a reversible and mathematical pendulum.
Ex.103 Determination of the linear expansion coefficient of solids.
Ex.104 Investigation of the moment of inertia
Ex.105 Determination of Young's modulus by the deflection method
Ex.106 Investigation of the uniformly accelerated motion using a computer measuring set
Ex.107 Determination of the dependence of the viscosity coefficient on temperature.
Ex.108 Determination of the stiffness modulus using the dynamic method.

ELECTROMAGNETISM

Ex.201 Determining the capacitance of a capacitor by means of relaxation vibrations
Ex.202 Investigation of the transformer
Ex.203 Determining the dependence of conductivity on temperature for semiconductors and conductors
Ex.204 Investigation of the influence of the magnetic field on a conductor with current
Ex.205 Determination of the Planck constant and output work based on photoelectric effect
Ex.206 Determination of ferromagnetic hysteresis loop by means of a hallotron
Ex.207 Calibration of the thermocouple
Ex.208 Measurement of the e/m ratio by means of deviations in the magnetic field

OPTICS

Ex.301 Determination of the refractive index of apparent and real thickness of the plates
Ex.302 Determination of focal length lenses from a lens pattern and the Bessel method
Ex.303 Determination of the diffraction grating constant
Ex.304 Optical emission spectra study
Ex.305 Determination of the refractive index of a liquid using an Abbe refractometer
Ex.306 Investigation of the polarization plane torsion caused by solutions using a polarimeter
Ex.307 Determination of the light efficiency of selected light sources
Ex.308 Determination of the radius of curvature of the lens using Newton rings

B. List of Physical constants

Table B.1. Physical constants

Name	Symbol	Value
Speed of light in vacuum	c	299 792 458(exact) m/s
Vacuum magnetic permeability	μ_0	$1.256\ 637\ 062\ 12(19) \cdot 10^{-6}$ N A^{-2}
Vacuum electric permittivity	ε_0	$8.854\ 187\ 8128(13) \cdot 10^{-12}$ F m^{-1}
Elementary charge	e	$1.602\ 176\ 634(\text{exact}) \cdot 10^{-19}$ C
Planck constant	h	$6.626\ 070\ 15(\text{exact}) \cdot 10^{-34}$ J Hz^{-1}
Avogadro constant	N_A	$6.022\ 140\ 76(\text{exact}) \cdot 10^{23}$ mol^{-1}
Electron mass	m_e	$9.109\ 383\ 7015(28) \cdot 10^{-31}$ kg
Proton mass	m_p	$1.672\ 621\ 923\ 69(51) \cdot 10^{-27}$ kg
Electron charge to mass quotient	$\frac{e}{m_e}$	$1.758\ 820\ 010\ 76(53) \cdot 10^{11}$ C kg^{-1}
Faraday constant	F	96 485.332 12...(exact) C mol^{-1}
Rydberg constant	R_∞	10 973 731.568 160(21) m^{-1}
Molar gas constant	R	8.314 462 618...(exact) J mol^{-1} K^{-1}
Boltzmann constant	k_B	$1.380\ 649(\text{exact}) \cdot 10^{-23}$ J K^{-1}
Stefan-Boltzmann constant	σ	$5.670\ 374\ 419... \cdot 10^{-8}$ W m^{-2} K^{-4}
Newtonian constant of gravitation	G	$6.674\ 30(15) \cdot 10^{-11}$ m^3 kg^{-1} s^{-2}
Standard acceleration of gravity	g	9.806 65 m s^{-2}

C. Tables

Table C.1. Some mechanical properties of solids at 20°C

Name of the solid	Density 10^3 kg m^{-3}	Young's modulus 10^{10} N m^{-2}	Stiffness modulus 10^{10} N m^{-2} rd^{-1}	Speed of sound 10^3 m s^{-1}	The lin. expansion coefficient α 10^{-6} K^{-1}
Bismuth	9.80	3.1	1.2	—	13.45
Tin	7.30	3.9-5.4	1.8	2.500	26.92
Zinc	7.04-7.19	3.4-13	2.6-4.6	3.700	13.45
Aluminum	2.70	6.2-7.3	2.2-2.7	5.104	25.5
Cadmium	8.66		2.3	2.307	—
Constantan	8.90	17	6.1	—	17.0
Fused Quartz	—	5.9	—	—	8-13
Copper	8.89	7.9-13	4.0-4.8	3.560	16.0-17.8
Brass (30% Zn)	8.44	10.3	4.2	3.500	18.9
Lead	11.34	1.4-1.7	0.64	1.277	29.4
Silver	10.49	6.9-7.9	2.4-2.9	2.610	18.8
Glass	2.6-5.9	4.9-7.9	1.7-3.0	5-6	9.0
Steel	7.83	21.5	8.15	4.990	6-13
Tungsten	18.6-19.3	35.4	13.2	—	4.5
Iron clean	7.85	—	—	5.130	—
Wrought iron	7.8-7.9	21.3	8.1	—	11.4

Table C.2. The thermal properties of solids at 20°C

Name of the solid	Specific heat 10^3 $Jkg^{-1}m^{-1}$	Melting temperature °C	Boiling point °C	Thermal conductivity $Jm^{-1}s^{-1}K^{-1}$
Bismuth	0.123	271.3	1560	—
Tin	0.226	231.9	2260	—
Zinc	0.384	419.0	907	64.8
Aluminum	0.896	569.7	2057	226
Cadmium	0.231	320.9	767	—
Constantan	0.41	1290	—	—
Fused Quartz		1710	—	12-67
Copper	0.385	1083	2336	384
Brass	0.388	910	—	85-109
Lead	0.128	327.4	1620	34.7
Silver	0.234	960.8	1950	825
Glass	0.832	800-1400	—	0.8-1.1
Steel	—	1400	—	48
Tungsten	0.144	3370	5900	199
Iron clean	—	1535	3000	67.2

Table C.3. The properties of liquids at 20°C

Name of liquid	Density 10^3 kg m^{-3}	Viscosity coefficient 10^{-3} $kgm^{-1}s^{-1}$	Specific heat 10^3 $Jkg^{-1}m^{-1}$	Melting temperature °C	Boiling point °C
Acetone	0.792	0.32	1.96	-95	57
Ethanol	0.791	1.2	2.38	-117.3	78.3
Methyl alcohol	0.788	0.6	2.51	-97.8	64.7
Benzene	0.878	0.65	1.72	5.48	80.2
Chloroform	1.480	0.56	0.98	-63.5	61.2
Ethyl ether	0.714	0.23	2.3	-117.6	34.6
Glycerine	1.260	1490	2.43	-17	291
Blood (37°C)	1.060	4	—	—	—
Castor oil	0.965	986	—	—	—
olive oil	0.910	84	—	2-6	300
Mercury	13.550	1.56	0.136	-38.87	356.9
Water	0.998	1	4.186	0.0	100.0

Table C.4. Specific resistance (in 0°C) and temperature coefficient of metals and alloys

Name of material	Specific resistance 10^{-8} Ωm	Temperature coefficient 10^{-3} K^{-1}	Name of material	Specific resistance 10^{-8} Ωm	Temperature coefficient 10^{-3} K^{-1}
Zinc	5.65	4.17	Newly Silver	30	0.35
Aluminum	2.5	4.6	Lead	19.2	4.28
Constantan	45	-0.05	Platinum	9.81	3.96
Manganin	43	0.02	Mercury	94.07	0.99
Copper	1.55	4.33	Silver	1.49	4.3
Brass	6.3	1.53	Tungsten	4.89	5.1
Nickel	6.14	6.92	Iron	8.6	6.51
Nickeline	43	0.23			

Table C.5. Dependence of thermoelectric force (ε) iron-constantine thermocouple on temperature difference (relative to 0°C)

Temperature °C	ε mV	Temperature °C	ε mV	Temperature °C	ε mV
0	0.00	350	19.32	700	39.30
50	2.66	400	22.07	750	42.48
100	5.40	450	24.82	800	45.72
150	8.19	500	27.58	850	49.00
200	10.99	550	30.39	900	52.29
250	13.97	600	33.27	950	55.25
300	16.56	650	36.24	1000	58.22

Table C.6. Dependence of thermoelectric force (ε) copper-constantine thermocouple on temperature difference (relative to 0°C)

Temperature difference °C	ε mV					
	-200	-100	0	100	200	300
0	-5.54	-3.35	0.00	4.28	9.29	14.86
10	-5.38	-3.06	0.40	4.70	9.82	15.44
20	-5.20	-2.77	0.80	5.23	10.44	16.03
30	-5.02	-2.46	1.20	5.71	10.90	16.62
40	-4.82	-2.14	1.67	6.20	11.46	17.22
50	-4.60	-1.85	2.03	6.70	12.01	17.82
60	-4.38	-1.47	2.47	7.21	12.57	18.42
70	-4.14	-1.11	2.91	7.72	13.14	19.02
80	-3.89	-0.75	3.36	8.23	13.71	19.63
90	-3.62	-0.38	3.81	8.76	14.28	—

Table C.7. Dependence of thermoelectric force (ε) chromium:nickel-nickel thermocouple on temperature difference

Temperature difference °C	ε mV				
	0	20	40	60	80
0	0.00	0.80	1.61	2.43	3.26
100	4.10	4.92	5.73	6.53	7.33
200	8.13	8.93	9.74	10.56	11.38
300	12.21	13.04	13.87	14.71	15.55
400	16.39	17.24	18.08	18.93	19.78
500	20.64	21.49	22.34	23.30	24.05
600	24.90	25.75	26.6	27.45	28.29
700	29.14	29.98	30.82	31.65	32.48
800	33.31	34.12	34.94	35.75	35.56
900	37.36	38.16	38.96	39.75	40.53
1000	41.31	42.08	42.86	43.62	44.38
1100	45.14	45.89	46.64	47.38	48.12
1200	48.85	49.57	50.29	51.00	51.71

Table C.8. The properties of ferromagnetic and ferrimagnetic bodies (μ_0 - initial magnetic permeability, ν_{max} maximum magnetic permeability, H_C - coercion, T_C - Curie temperature)

Material	μ_0	μ_{max}	H_0 Am^{-1}	T_C K
78 Permalloy (78.5% Ni)	8000	100000	4	473
$CoFe_2O_4$	1	1	52000	768
Ferrocobalt (35% Co)	1000	27000	16-60	1253
Cobalt (99% Co)	70	250	800	1393
MnBi (20% Mn, 80% Bi)	—	—	260000	633
Nickel (99% Ni)	110	600	55	631
Silicon steel (4% Si)	500	7000	40	963
Mild steel (0.2% C)	120	2000	143	1043
Supermalloy (5% Mo, 79% Ni)	100000	1000000	0.16	673
Iron oxide (Fe_3O_4)	70	70	—	858
Pure iron (99.95% Fe)	10000	100000	4	1053
Technical irons (99.8% Fe)	150	5000	80	1053

Table C.9. Refractive index (n) relative to air (at 15 °C for the yellow sodium line)

Material	n	Material	n
Acetone	1.360	Fused Quartz	1.458
Ethanol	1.360	Paraffin oil	1.440
Methyl alcohol	1.330	Ordinary glass	1.518
Benzene	1.504	Crown glass	1.525
Amber	1.546	Flint glass	1.569
Carbon tetroxide	1.464	water	1.333
Glycerine	1.470	Gelatine	1.530

Table C.10. The wavelength of the spectral lines of some elements
(vs - very strong, s - strong, m - medium, w - weak, vw - very weak)

λ (nm) / Intens.	λ (nm) / Intens.	λ (nm) / Intens.	λ (nm) / Intens.
H - hydrogen	O - oxygen	Kd - cadmium	Na - sodium
410.2 vw	394.7 vw	467.8 s	589.6 m
434.0 w	436.8 vw	480.0 w	589.0 m
486.1 m	543.6 vw	508.5 vw	Zn - zinc
656.3 vs	557.7 vw	515.5 s	463.0 w
He - helium	595.9 vw	643.8 s	468.0 w
388.8 m	610.6 vw	738.4 vw	472.2 w
438.8 vw	615.6 m	783.5 vw	481.2 w
447.1 m	645.6 w	Cu - cooper	518.2 w
471.3 w	700.2 w	402.3 vw	636.2 w
492.2 w	725.4 w	406.3 vw	Kr - krypton
501.6 s	Ne - neon	427.5 vw	427.4 s
504.8 vw	534.1 vw	437.8 vw	432.0 s
587.6 vs	540.1 vw	458.7 vw	437.6 s
667.8 vs	576.4 vw	515.3 w	445.4 m
706.5 vs	585.2 vs	521.8 vw	450.2 m
728.1 m	594.5 m	570.0 vw	556.2 m
Li - lithium	603.0 w	578.2 vw	557.0 vs
610.4 s	609.6 s	Ar - argon	587.1 vs
670.8 m	614.3 s	420.1 w	645.6 vw
N - nitrogen	621.7 w	425.9 w	Hg - mercury
410.0 vw	626.6 m	641.6 vw	365.0 s
484.7 vw	633.4 m	667.7 w	404.7 m
491.5 vw	640.2 vs	696.5 vw	407.8 vw
528.1 vw	650.7 s	727.3 w	435.8 vs
575.2 w	659.9 m	750.4 vs	491.6 vw
583.0 w	667.8 m	751.4 vs	512.8 vw
600.0 vw	671.7 w	763.5 m	546.1 vs
642.1 vw	692.9 m	794.8 vw	577.0 s
700.2 v	702.4 m	800.6 s	579.1 s
725.4 v	717.4 vw	811.5 w	623.4 vw
746.8 m	724.5 w	840.8 w	671.6 vw
	K -potasium	912.3 vw	690.7 vw
	393.4 w		708.2 vw
	396.8 w		772.9 vw
	645.7 vs		

(for more spectra lines visit NIST Atomic Spectra Database Lines Form https://physics.nist.gov/PhysRefData/ASD/lines_form.html [10] or click to use *Spektruś* program)

D. REGULATIONS of the Physics Laboratory

Regulations of the Physics Laboratory at the Poznań University of Technology

I. GENERAL PROVISIONS/REGULATIONS

1. The aim of the classes at the Physics Laboratory is to experimentally check basic physical laws, to familiarize yourself with instruments, measuring technique and analysis of measurement results. The above goals are achieved by performing experiments.
2. Depending on the field of study, 2 or 3 hours (90 or 135 minutes) are allocated to perform one exercise (experiment).
3. Students perform exercises in the Physics Laboratory in teams of maximum two people. In special cases, a larger number of people can form a team.
4. Each experience has its own position in the workshop, where most of the necessary equipment is located. If necessary, before starting the exercise, rent in the technical room (room 221A) small measuring equipment, such as: calipers, micrometer screws, measures, ruler, stoppers, etc.
5. Electrical devices can be connected to a power source and switched off after the exercise, only after obtaining the consent of the lecturer and in his presence.
6. The student bears full material responsibility for damage to the devices caused by the student.
7. During classes you must not leave the station without the consent of the teacher conducting the exercises.

II. PREPARATION AND EXECUTION OF THE EXPERIMENT

1. You should be prepared for each exercise, i.e.
   a) master the necessary theoretical knowledge about experience,
   b) get acquainted with the course of the exercise and the principle of operation of the instruments used. NOTE: The appropriate topic of the exercise should be selected based on the recommendations given on the information board or on the website of the Physics Laboratory.
2. The doubts arising during the experiment are resolved by the teacher conducting the classes. He also checks the student's preparation for classes.
3. During the experiments, enter in the report:

   a) measurement results included in the tables (direct readings) with appropriate units,
   b) uncertainty of measured quantities (reading accuracy).
4. Before leaving the workshop, written confirmation must be obtained by the teacher conducting the exercises of the results in the report.
5. The instrument set can only be dismantled after the results have been confirmed. We disconnect the electrical circuits by switching off the power source. After completing the exercise, you should organize your position.
6. After taking the measurements, the results should be developed, i.e.
   a) using the measurement results, calculate the searched physical quantities and calculate the units,
   b) calculate the measurement uncertainties of the determined quantities and compile the results,
   c) prepare charts in accordance with the rules contained in the script (S. Szuba - Physics laboratory exercises [1]),
   d) present conclusions regarding the given experiment and, if possible, compare the obtained results with the literature data.
   e) NOTE: Although students do exercises in teams, they are assessed individually.

III. COMPLETION OF CLASSES AND SEMESTER CREDIT

1. Conditions for passing the exercise are as follows:
   a) positive assessment of theoretical knowledge,
   b) correct measurements,
   c) correct processing of measurement results,
   d) submitting the report before proceeding to the next exercise.
2. The above factors influence the final assessment of the exercise.
3. Completion of laboratory exercises takes place in the last week of classes after completing the exercise and giving a shortened (as required by the teacher) report.
4. A prerequisite for passing the laboratory classes is passing a positive grade (minimum satisfactory) of at least 85% of all classes provided in a given semester.
5. Unjustified absences are tantamount to failing the exercise (unsatisfactory) and lower the final grade.
6. Absences due to reasons beyond the control of the student should be excused for the teacher during the first class after the absence. Otherwise, your absence will not be excused.
7. In the case of an excused absence, the student has the right to do the exercise on the date agreed with the teacher, when the given measuring position is free. Completing the completed exercise requires meeting all the requirements listed in section III.1, however, the number of completed exercises cannot be greater than 1/3 of the planned exercises in the semester.
8. In the event that a student has completed at least 85% of all exercises and has not received a credit within the prescribed period, he / she has the right to proceed with a corrective and commission credit in accordance with the study regulations.

E. REGULATIONS - order and OHS

Order regulations and OHS regulations in force in the Physics Laboratory

OHS - "Occupational Health and Safety"

I. GENERAL PROVISIONS/REGULATIONS

1. The order regulations of the Physical Laboratory of the Faculty of Technical Physics of the Poznań University of Technology set out the basic formal regulations, rules of conduct and OHS requirements when using the laboratory resources of the Faculty of Technical Physics in the field of conduct on the premises and in relation to the equipment of the Laboratory.
2. The Laboratory operates according to the rules set by its supervisor and approved by the Dean of the Faculty.
3. All persons staying at the Laboratory are obliged to comply with the provisions contained in these regulations, general health and safety regulations, fire protection and not to disturb the order in the rooms.
4. It is forbidden to enter the rooms of the Workwear in outerwear, bring large bags, backpacks, suitcases. Outerwear (especially wet), large luggage, umbrellas, etc. should be left in the cloakroom.
5. It is forbidden to bring and consume meals and drinks (in any form) in the Laboratory.

II. SPECIFIC PROVISIONS

Students participating in the laboratory exercises of the Physics Laboratory are exposed to electric shock and harmful effects of laser and microwave radiation. Therefore, students are required to exercise extreme caution, and in particular should:

1. Familiarize yourself with the location of the main switch and other switches enabling immediate disconnection of work stations from power sources.
2. Organize the workplace before taking measurements and observe order and cleanliness.
3. Familiarize yourself with the specifications of the equipment and devices that will be used to make measurements.
4. Connect measuring systems to the power source only with the consent of the teacher conducting the class.
5. Be ready to immediately turn off the power supply to the systems during measurements.

6. Be careful when changing the ranges of measuring instruments - do not disconnect power supply in the tested circuit when changing the measuring range.
7. Make any configuration changes to the electrical connections in the tested measuring system only after disconnecting the power supply and under the supervision of the instructor.
8. Keep in mind the capacitors installed in the systems and the electrical potential accumulated in them, which may pose a risk of electric shock even after disconnecting the voltage supplying the tested system.
9. Consider the impact of other workplaces on safety when making measurements at your workstation.
10. Perform exercises only at the position indicated by the teacher. Do not use equipment other than the one assigned to perform the exercise.
11. Avoid arbitrary switching to other test stands during measurements.
12. When moving around the workshop, be extremely careful not to damage adjacent workstations.
13. In the event of an electric shock, disconnect the injured person and the station from the power source and proceed to rescue operations described in a separate "first aid instruction".
14. Absolutely protect your eyes against laser radiation and make sure that the reflected laser radiation does not hit the eyes of an outsider.
15. Take special care and do not approach closer than 20 cm from the outlet of the microwave transmitter (klystron).
16. Supervise and control the apparatus throughout the entire exercise.
17. Inform the teacher conducting the exercises about an incident threatening health and life. Students in threatened rooms not designated for rescue operations are obliged to leave them without delay.

III. Additional regulations related to safety at the Physics Laboratoryduring the pandemic (CC

1. Entering the laboratory room is a simultaneous declaration of good health.
2. Students and academic teachers are required to disinfect their hands both before and immediately after conducting classes in the 1st Physical Laboratory.
3. During laboratory classes, people in the room should:
   a) wear protective masks or visors,
   b) keep social distance as much as possible,
   c) avoid touching your mouth, nose and eyes.
4. Students are required to minimize movement around the room and limit personal items on work tables to the necessary minimum.
5. Laboratory exercises should end 15 minutes before the scheduled end of classes in order to disinfect the tops of laboratory tables and ventilate the room.
6. The laboratory leader decides on the scope of the course of the abbreviated form of the exercise.

7. At the request of the teacher, students can send reports from the exercises in electronic form, and then review them by the teachers in the same form. In this situation, the reports should be archived in electronic form for a period of 1 year

F. Template of Report

Ex. No 204	Date DD.MM.YYYY	Faculty Electrical Eng.	Field of study AC&R	Lab. Group 2
Academic teacher		Preparation	Execution	Final Grade

Title: ***Investigation of the influence of a magnetic field on a conductor with current***

1. **Introduction**
2. **Results of Measurements**
3. **Calculations**
4. **Estimation of errors**
5. **Plots/graphs**
6. **Final results**
   (including correct rounding up values)
   for example $e/m_e = (1.758 \pm 0.005) \times 10^{11}$ [C kg^{-1}]
   or $e/m_e = 1.758(5) \times 10^{11}$ [C kg^{-1}]
7. **Summary/Conclusions**

G. Templates of plots (made in Phyton (x.y) and mathplotlib)

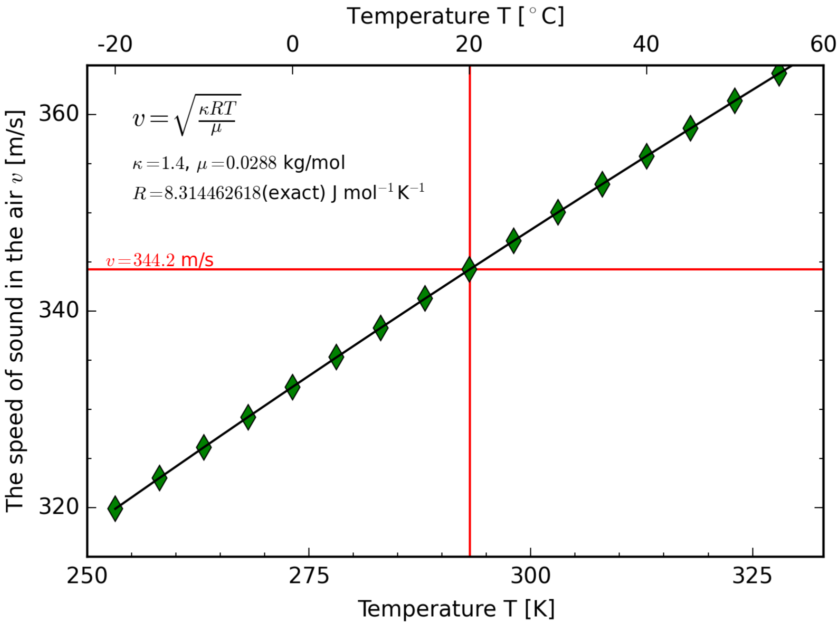


Figure G.1. Ex.101. Plot 1. Theoretical calculated the speed of sound as a function of temperature. Red lines shows the theoretical values for temp.=20 °C.

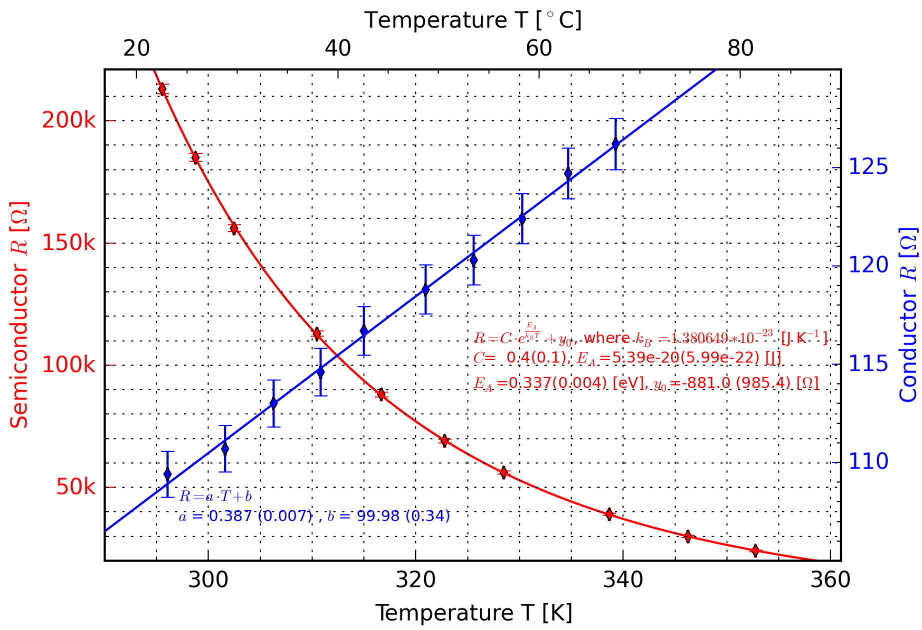


Figure G.2. Ex.203. Plot 1. An example of plot $R = f(Temp.)$ for both conductor and semiconductor. See the value of E_A is determined.

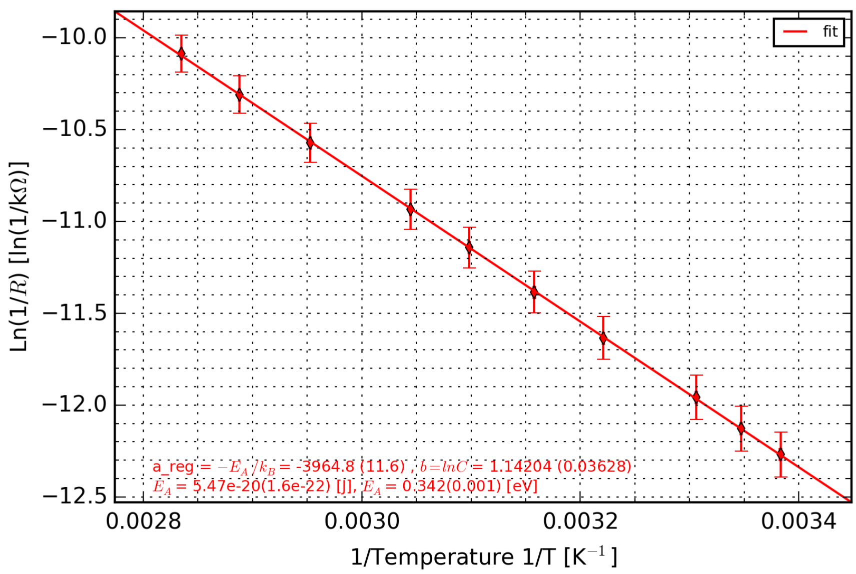


Figure G.3. Ex.203. Plot 2. An example of plot $ln(1/R) = F(1/Temp.)$, see the value of E_A is determined.

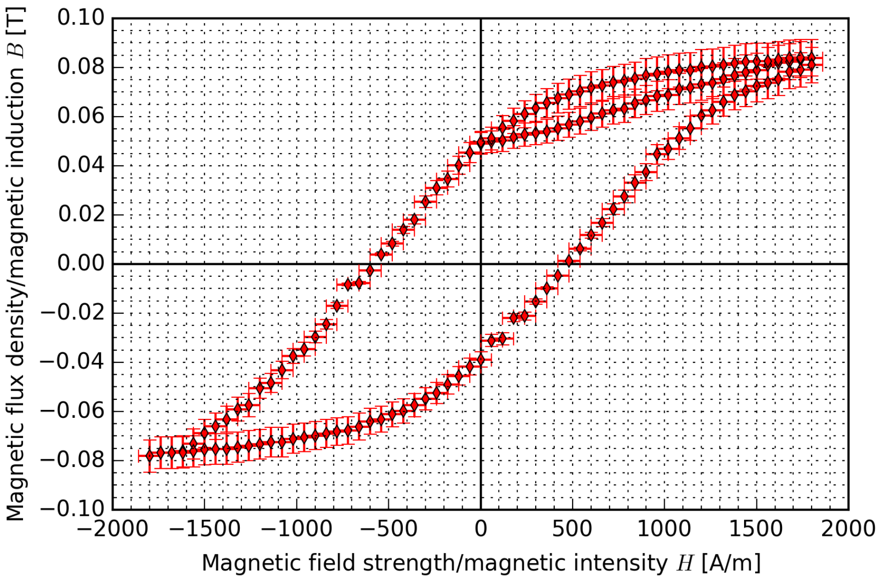


Figure G.4. Ex.206. Plot 1. An example of plot $B = F(H)$ the hysteresis loop for ferromagnetic.

H. Specifications of measuring instruments

Multimeter MASTECH MY70/74

Table H.1. Multimeter MASTECH MY70/74 - VOLTAGE MEASUREMENT, DC

measurement range	resolution	measurement accuracy
200 mV	0,1 mV	0,5% read + 2 digits
2 V	1 mV	0,5% reading + 2 digits
20 V	10 mV	0,5% reading + 2 digits
200 V	100 mV	0,5% reading + 2 digits
600 V	1 V	0,8% reading + 2 digits

Table H.2. Multimeter MASTECH MY70/74 - VOLTAGE MEASUREMENT, AC

measurement range	resolution	measurement accuracy
200 mV	0,1 mV	1,2% reading + 3 digits
2 V	1 mV	0,8% reading + 3 digits
20 V	10 mV	0,8% reading + 3 digits
200 V	100 mV	0,8% reading + 3 digits
600 V	1 V	1,2% reading + 3 digits

Table H.3. Multimeter MASTECH MY70/74 - RESISTANCE MEASUREMENT

measurement range	resolution	measurement accuracy
200 Ω	0,1 Ω	0,8% reading + 3 digits
2 kΩ	1 Ω	0,8% reading + 2 digits
20 kΩ	10 Ω	0,8% reading + 2 digits
200 kΩ	100 Ω	0,8% reading + 2 digits
2 MΩ	1 kΩ	0,8% reading + 2 digits
20 MΩ	10 kΩ	1,0% reading + 2 digits
200 MΩ	100k Ω	6,0% reading + 10 digits

Table H.4. Multimeter MASTECH MY70/74 - MEASUREMENT OF CURRENT, DC

measurement range	resolution	measurement accuracy
20 μA	0,01 μA	2,0% reading + 5 digits
200 μA	0,1 μA	0,8% reading + 1 digit
2 mA	1 μA	0,8% reading + 1 digit
20 mA	10 μA	0,8% reading + 1 digit
200 mA	0,1 mA	1,5% reading + 1 digit
10 A	10 mA	2,0% reading + 5 digits

Table H.5. Multimeter MASTECH MY70/74 - MEASUREMENT OF CURRENT, AC

measurement range	resolution	measurement accuracy
20 μA	0,01 μA	2,0% reading + 5 digits
200 μA	0,1 μA	1,0% reading + 5 digits
2 mA	1 μA	1,0% reading + 5 digits
20 mA	10 μA	1,0% reading + 5 digits
200 mA	0,1 mA	1,8% reading + 5 digits
10 A	10 mA	3,0% reading + 7 digits

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